

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: CVX5443 Structural Analysis
Academic Year	: 2024/25
Date	: 12 th March 2026
Time	: 0930-1230hrs
Duration	: 3 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
5. Answer for each question should commence from a new page.
6. This is Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

QUESTION 1

- (i) Briefly describe “**stress state in a 3D dimensional body**” using neat sketches (3 Marks)
- (ii) Strain tensor for a homogenous, isotropic material is given below.

$$\varepsilon_{ij} = \begin{bmatrix} -100 & 0 & -40 \\ 0 & 10 & 0 \\ -40 & 0 & 20 \end{bmatrix} \text{ in micro-strains}$$

- (a) Write six independent strain components (3 Marks)
- (b) Determine three strain invariants (4 Marks)
- (c) Determine three principal strains (5 Marks)
- (d) Determine three principal stresses if the elastic modulus and Poison’s ratio are 200 GPa and 0.3; respectively. (5 Marks)

QUESTION 2

- (i) State **St. Venant’s Principle** and briefly explain its importance in structural analysis. (4 Marks)
- (ii) A simply supported beam of length ‘ l ’ is subjected to a uniformly distributed load ‘ w ’. The beam has a rectangular cross section of breath ‘ b ’ and depth ‘ d ’.
- (a) stress function given below is a possible stress function. (4 Marks)

$$\phi = \frac{A}{6} \left(l^2 y^3 - x^2 y^3 + \frac{1}{5} y^5 \right) + Bx^2 + Cx^2 y$$

- (b) Determine constants A, B and C applying the relevant boundary conditions of the above simply supported beam. (3 Marks)
- (a) Determine three stress components. (6 Marks)
- (b) Briefly explain the limitations of the above stress function. (3 Marks)

QUESTION 3

- (i) Explain three characteristics of statically indeterminate structures (4 Marks)
- (ii) A continuous beam (ABCD) is shown in Figure Q3. Flexural rigidities of members AB and CD are equal to EI and member BC is 2EI. Uniformly distributed loads ($2W$), ($4W$) are acting on members, AB and BC, respectively. There is a concentrated load (Wl) in member CD.

- Determine the degree of the statical indeterminacy of the beam. (3 Marks)
- Draw a released structure. (3 Marks)
- Determine the flexibility matrix for the drawn released structure. (4 Marks)
- Determine bending moments at B and C. (6 Marks)

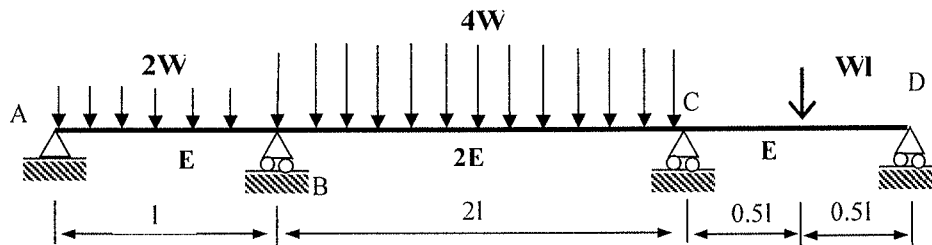


Figure Q3

QUESTION 4

A portal frame structure shown in Figure Q4. All members have same flexural rigidity (EI). You can neglect the axial deformation.

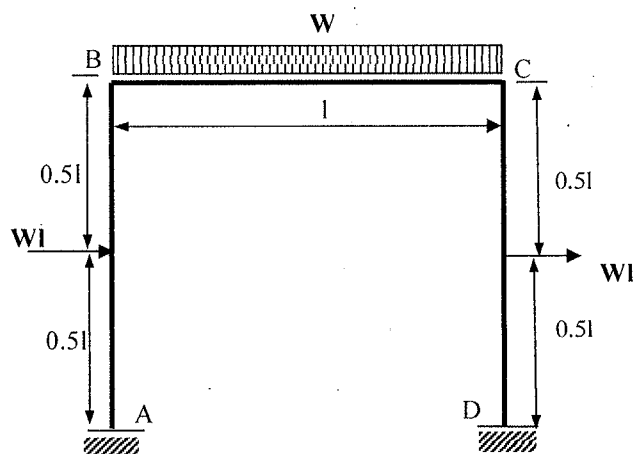


Figure Q4

- Find Kinematic indeterminacy of the structure (2 Marks)
- Draw the structure with independent nodal displacements (2 Marks)
- Determine the flexibility matrix of the structures. (4 Marks)
- Find the free nodal displacements at B using the displacement method. (6 Marks)
- Using above results, determine the bending moment at B. (6 Marks)

QUESTION 5

- (i) Briefly describe the photo-elasticity method used in experimental stress analysis methods. (2 Marks)
- (ii) Briefly explain why **out of balance method** is preferred over **null method** in experimental stress measurements of a steel railway truss bridge due to train movements. (3 marks)
- (iii) The stress state of a certain steel components was determined using a strain rosette as shown in Figure Q5. Due to the loadings, strain gauges gave strain values as $\epsilon_a = 60 \mu\epsilon$, $\epsilon_b = 100 \mu\epsilon$, $\epsilon_c = 120 \mu\epsilon$.

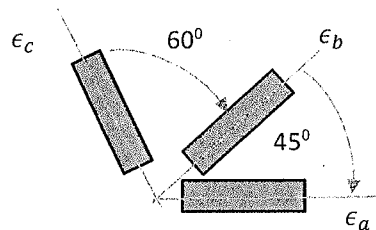


Figure Q5

- (a) Draw the Mohr's circle for strains (4 marks)
- (b) Determine the in-plane principal strains. (5 marks)
- (c) Determine the principal stress, if the material is steel ($E = 200\text{GPa}$, $\nu = 0.3$) (4 marks)
- (d) Determine the von Mises stress (2 marks)

QUESTION 6

- (i) Briefly describe three conditions used in plastic analysis of structures. (2 Marks)
- (ii) A two-bay frame structure is shown in Figure Q6. Dimensions and plastic moments of the columns and beam are given in the figure.
 - (a) Draw possible locations of plastic hinge formations. (2 Marks)
 - (b) Draw elementary failure mechanisms. (2 Marks)
 - (c) Determine load factors for each elementary failure mechanism. (6 Marks)
 - (d) Determine the most probable failure mechanism by combining elementary failure mechanisms. (6 Marks)
 - (e) Explain how you can ensure the unique solution. (2 Marks)

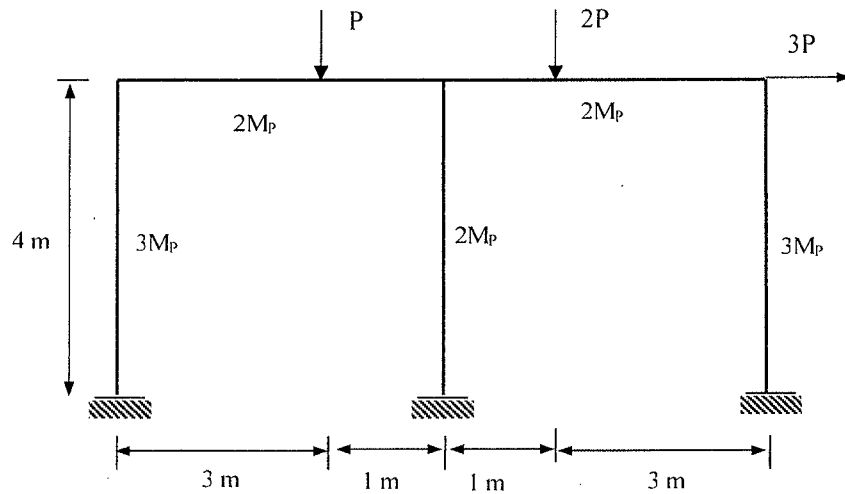


Figure Q6

QUESTION 7

The governing equation for symmetrical bending of solid circular plates can be expressed as given below

$$\left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right] \left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right] w = \frac{q(r)}{D}$$

All the terms have their normal meanings.

(i) Write two conditions that are needed to satisfy symmetrical bending (4 Marks)

(ii) Show that the above equation can be simplified to the following form

$$\frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] \right\} = \frac{q(r)}{D} \quad (4 \text{ Marks})$$

(iii) Show that for a solid plate the deflection is given by the following expression.

$$w = \frac{q_0 r^4}{64D} + C_2 \frac{r^2}{4} + C_4 \quad \text{where } C_2 \text{ and } C_4 \text{ are constants.} \quad (4 \text{ Marks})$$

(iv) Hence show that for a clamped plate deflection is given by

$$w = \frac{q_0}{64D} (a^2 - r^2)^2 \quad (8 \text{ Marks})$$

QUESTION 8

(i) Briefly describe following definitions used in the membrane theory of thin shells.

(a) Middle surface

(b) Normal plane

(c) Gaussian curvature

(3 Marks)

(ii) A spherical dome with radius ' a ' is supported with a uniformly distributed load ' w ' on its plane as shown in Figure Q8. The shell thickness is ' h '.

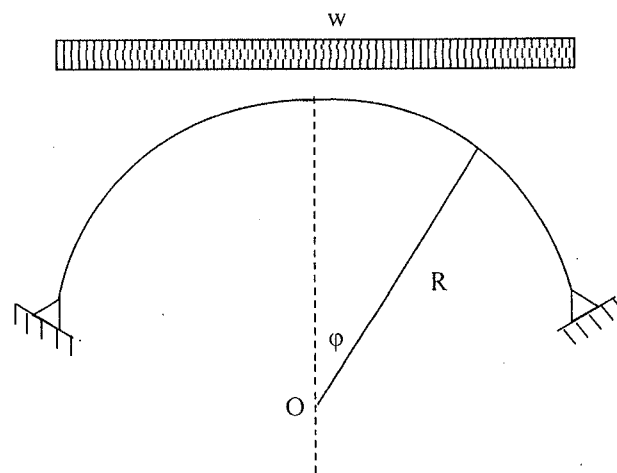


Figure Q8

(a) Derive expressions for meridinal and hoop stresses in the sphere.

(9 Marks)

(b) Show that the practical limitation of angle ϕ is 45° , if the dome is made of concrete.

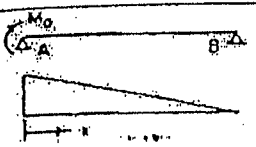
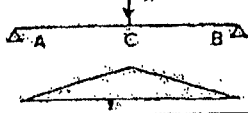
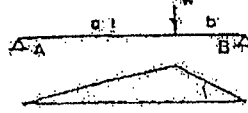
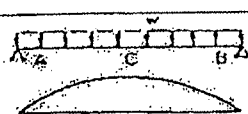
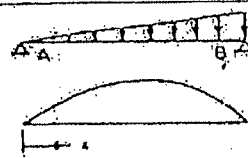
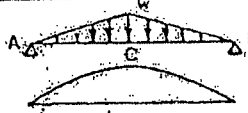
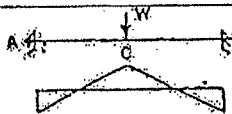
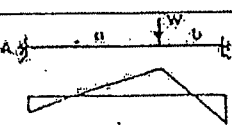
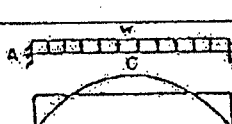
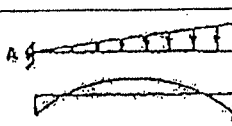
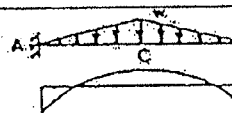
(5 Marks)

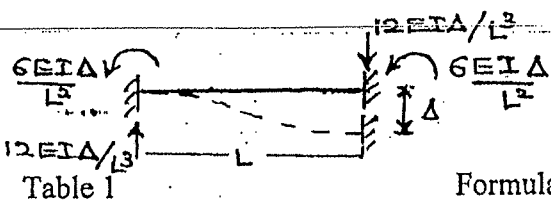
(c) If angle ϕ is to be increased more than 45° , logically describe alterations that you would propose in the design.

(3 Marks)

Table 1

Formulas for Beams

Structure	Shear 	Moment 	Slope 	Deflection 
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{\max} = 0.062 \frac{M_o L^2}{EI}$ at $x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{\max} = 0.064WL^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{\max} = 0.00652 \frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5WL^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{\max} = 0.00131 \frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$



Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0	M_0	$\theta_A = \frac{M_0 L}{EI}$	$Y_A = \frac{M_0 L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_0}{2L}$	$M_B = -\frac{M_0}{2}$	$\theta_A = -\frac{M_0 L}{4EI}$	$Y_{\max} = \frac{M_0 L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_0}{2L}$	$M_B = -\frac{3WL}{16}$ $M_C = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{\max} = 0.00962 \frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2-a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Y_0 = \frac{Wa^2b^2}{12EIL^3}(3L+a)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{\max} = 0.0054 \frac{WL^4}{EI}$ at $x = 0.42L$
	$S_A = +\frac{WL}{10}$	$M_{\max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{\max} = 0.00239 \frac{WL^4}{EI}$ at $x = 0.44L$
	$S_A = +\frac{11WL}{40}$	$M_{\max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{\max} = 0.00305 \frac{WL^4}{EI}$ at $x = 0.40L$