0102

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	:	Bachelor of Science Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	:	**EEX5360- Signals and Systems**
Academic Year	:	2024/25
Date	:	08th March 2026
Time	:	0930-1230h
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Five (5)** questions in **Five (5)** pages.
3. Answer **all** questions.
4. All questions carry equal marks.
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test (CBT).

Question 1 **(20 Marks)**

A passive electrical network is shown in the following *Figure Q1*. x(t) is a DC source and $y(t)$ is the current flowing through the network. The initial conditions are given as $y_0(0) = 0$ and $\dot{y}_0(0) = 1$.

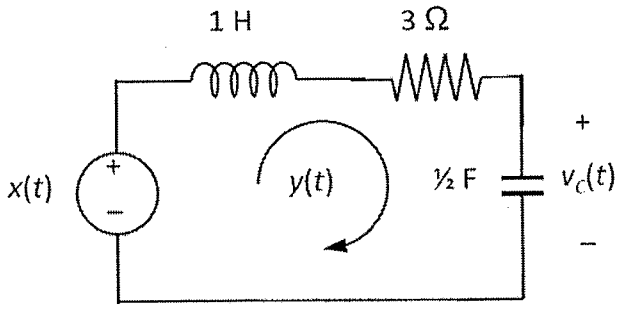


Figure Q1

a) Write an equation for $x(t)$ in terms of $y(t)$. (02 marks)

b) Write an equation for the zero-input response of the circuit using the differential equations. (04 marks)

c) Hence obtain the characteristic equation of the system. (04 marks)

d) Write the zero-input response ($y_0(t)$) of the circuit. (05 marks)

e) Determine the unit impulse response of the system. (05 marks)

Question 2 **(20 Marks)**

i. A Discrete time signal $x[n]$ is shown in *Figure Q2*. The signal is zero outside the range, $-2 \leq n \leq 1$.

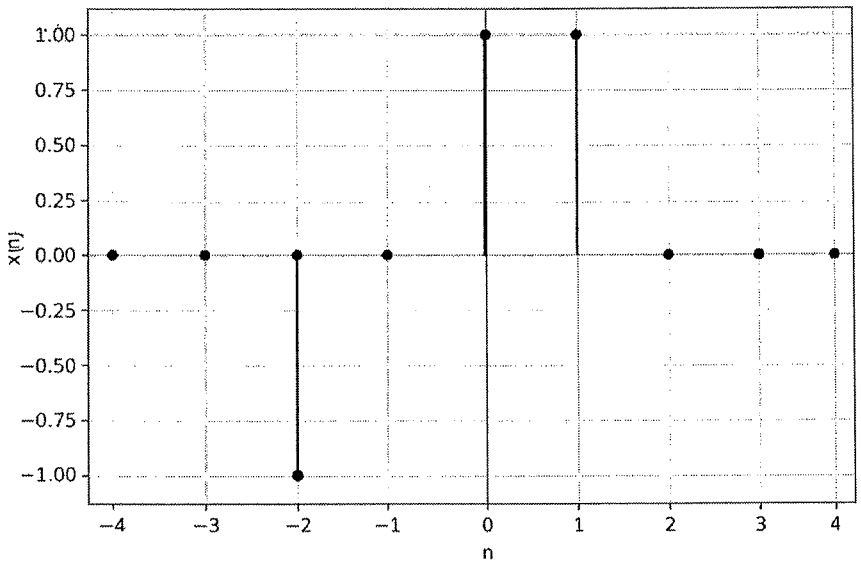


Figure Q2

(a) Sketch the signal $y_1[n] = x[n-1]$. (03 marks)

(b) Sketch the signal $y_2[n] = x[2n+2]$. (03 marks)

(c) Assume that the $x[n]$ can be written as the sum of an even part $x_e[n] = x_e[-n]$ and the odd part $x_o[n] = -x_o[-n]$. Prove that $x_o[n]$is unique and sketch the $x_o[n]$. (05 marks)

ii. A continuous time system H has an input $x(t)$ and output $y(t)$ related by,

$$y(t) = x(-2t+1) + 1$$

(a) Is the system H Causal? Justify your answer. (03 marks)

(b) Is the system H linear? Justify your answer. (03 marks)

(c) Is the system H time invariant? Justify your answer. (03 marks)

Question 3 **(20 Marks)**

A passive RLC circuit with a current source is shown in *Figure Q3.*

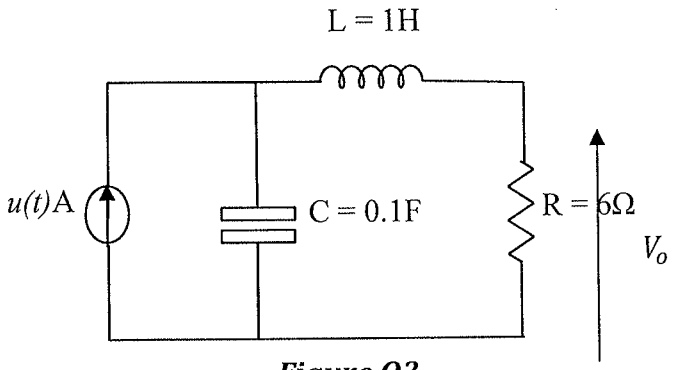


Figure Q3

(a) Transform the circuit elements from time domain to Laplace domain. (06 marks)

(b) Hence, draw the Laplace domain equivalent circuit for *Figure Q3.* (04 marks)

(c) Find the output voltage (V_0) across the resistor. (10 marks)

Question 4 **(20 Marks)**

(a) A sequence is defined as, $f(k) = \left(\frac{1}{2}\right)^k$. Determine the Z-transformation of the shifted sequence $f[k+1]$. (08 marks)

(b) Using the Z Transformation, solve the following second order differential equation.

$$y[k+2] - 5y[k+1] + 6y[k] = 0$$

Subjected to the initial conditions, $y(0) = 1$ and $y(1) = 3$. (12 marks)

Question 5 **(20 Marks)**

Consider the block diagram below in Figure Q5, where period, $T = 0.005\ s$.

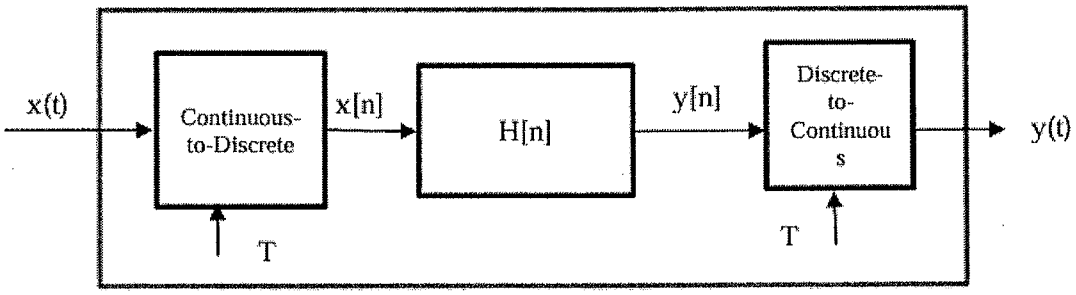


Figure Q5

The continuous-time input signal is

$$x(t) = 10\cos(200\pi t) + 5\cos(600\pi t)$$

Here:

- $x(t)$and $y(t)$represent the continuous-time input and output of the system, respectively.
- $x[n]$and $y[n]$represent the corresponding discrete-time input and output signals.
- $H[n]$denotes the discrete-time impulse response of the system.

Assume that $H[n]$ is an ideal low pass filter with the cutoff frequency $\omega_c = \frac{2\pi}{5}$ rad/sec.

(a) State and plot the continuous time Fourier transform $X(j\omega)$ of $x(t)$. (05 marks)

(b) State and plot the discrete time Fourier transform $X(e^{j\omega})$ of $x[n]$. (05 marks)

(c) State and plot the continuous time Fourier transform $X(e^{j\omega})$ of $y[n]$. (05 marks)

(d) State and plot the continuous time Fourier transform $Y(j\omega)$ of $y(t)$. (05 marks)

Formula sheet

1. Laplace Transformation-Important Formulas

$$L\{e^{-t}\} = \frac{1}{s+1}$$

$$L\{\delta(t)\} = 1$$

$$L\{\delta(t-a)\} = e^{-as}$$

1.1.1 Ordinary Differential Equation Formulas

$$L\left\{\frac{d^2x}{dt^2}\right\} = s^2X(s) - sx(0) - \dot{x}(0)$$

$$L\left\{\frac{dx}{dt}\right\} = sX(s) - x(0)$$

1.1.2 Second Shift Theorem

$$L[f(t-a)+u(t-a)] = e^{-as}F(s)$$

1.2 Inverse of the Second Shift Theorem

$$L^{-1}[fe^{-as}F(s))] = f(t-a)+u(t-a)$$

2. Z-Transformation pairs

Z Transform Pairs and Properties

Z Transform Pairs		
Time Domain *	**Z Domain**	
	z	**z^{-1}**
δ[k] (unit impulse)	1	1
γ[k] † (unit step)	$\frac{z}{z-1}$	$\frac{1}{1-z^{-1}}$
a^k	$\frac{z}{z-a}$	$\frac{1}{1-z^{-1}a}$
e^{-bTk}	$\frac{z}{z-e^{-bT}}$	$\frac{1}{1-z^{-1}e^{-bT}}$
k	$\frac{z}{(z-1)^2}$	$\frac{z^{-1}}{(1-z^{-1})^2}$
sin(bk)	$\frac{z\sin(b)}{z^2-2z\cos(b)+1}$	$\frac{z^{-1}\sin(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
cos(bk)	$\frac{z(z-\cos(b))}{z^2-2z\cos(b)+1}$	$\frac{1-z^{-1}\cos(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
$a^k\sin(bk)$	$\frac{az\sin(b)}{z^2-2az\cos(b)+a^2}$	$\frac{az^{-1}\sin(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$
$a^k\cos(bk)$	$\frac{z(z-a\cos(b))}{z^2-2az\cos(b)+a^2}$	$\frac{1-az^{-1}\cos(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$

Z Transform Properties	
Property Name	**Illustration**
Linearity	$af_1[k]+bf_2[k] \xleftrightarrow{Z} aF_1(z)+bF_2(z)$
Left Shift by 1	$f[k+1] \xleftrightarrow{Z} zF(z)-zf[0]$
Left Shift by 2	$f[k+2] \xleftrightarrow{Z} z^2F(z)-z^2f[0]-zf[1]$
Left Shift by n	$f[k+n] \xleftrightarrow{Z} z^nF(z)-z^n\sum_{k=0}^{n-1}f[k]z^{-k} = z^n\left(F(z)-\sum_{k=0}^{n-1}f[k]z^{-k}\right)$
Right Shift by n	$f[k-n] \xleftrightarrow{Z} z^{-n}F(z)$
Multiplication by time	$kf[k] \xleftrightarrow{Z} -z\frac{dF(z)}{dz}$
Scale in z	$a^kf[k] \xleftrightarrow{Z} F\left(\frac{z}{a}\right)$
Scale in time	$f\left[\frac{k}{n}\right] \xleftrightarrow{Z} F(z^n);$ n is an integer, $n \geq 1$
Convolution	$f_1[k]*f_2[k] \xleftrightarrow{Z} F_1(z)F_2(z)$
Initial Value Theorem	$f[0] = \lim_{z\to\infty} F(z)$
Final Value Theorem (if final value exists)	$\lim_{k\to\infty} f[k] = \lim_{z\to 1}(z-1)F(z)$

4. Fourier Transform pairs

X(t)	X(jω)
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$e^{-at}u(t)$	$\frac{1}{j\omega+a}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(j\omega+a)^n}$
$sin\omega_0 t$	$j\pi\delta(\omega+\omega_0) - j\pi\delta(\omega-\omega_0)$
$cos\omega_0 t$	$\pi\delta(\omega+\omega_0) - j - \pi\delta(\omega-\omega_0)$
$x(t-t_0)$	$X(j\omega)e^{-j\omega t_0}$

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