0187

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	:	Bachelor of Science Honours in Engineering
Name of the Examination	:	Final Examination
Course Code anmd Title	:	**MHZ5554 – Engineering Mathematics IV**
Academic Year	:	2024/25
Date	:	03rd March 2026
Time	:	1330-1630h
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. Part A is compulsory.
3. Part B contains 07 (seven) questions. Answer **only 05 (five)** from part B.
4. All the symbols are in standard notation unless they are defined.
5. All symbols are in standard notation and state any assumption that you made.
6. All questions carry equal marks.
7. Answer for each question should commence from a new page.
8. This is a Closed Book Test (CBT).
9. Non-programable calculators are allowed.

Part A

1. Evaluate $\iint dA$ over the region in the first quadrant for which $-x + y \leq 2, x = 2$.

   (10 marks)

2. Let $\boldsymbol{F} = zy\boldsymbol{i} + xz\boldsymbol{j} + xy\boldsymbol{k}$. Find $\nabla \cdot \boldsymbol{F}$ and hence, evaluate $\iint (\boldsymbol{F} \cdot \boldsymbol{n}) dA$. *(10 marks)*

3. Let $\boldsymbol{F} = (x^2 + 1)\boldsymbol{i} + (y^2 + 1)\boldsymbol{j} + (z^2 + 1)\boldsymbol{k}$. Find $\nabla \times \boldsymbol{F}$ and hence evaluate $\int \boldsymbol{F} \cdot d\boldsymbol{r}$. *(10 marks)*

4. Let $z \in \mathbb{C}$. Determine the nature of singularity of the function $f(z) = \frac{e^{2z}-1}{z}$.

   (10 marks)

5. Write down the Taylor series expansion of the function $f(z) = \frac{1}{3+z}, z \neq -\frac{3}{4}$ about $z = -1$. *(10 marks)*

6. Sketch the image of the line $x = 1$ under the mapping $w = z^2$. *(10 marks)*

7. Find the Fourier Sine transformation of,

$$f(t) = \begin{cases} 2 & ; 0 < t < 1 \\ 0 & ; t > 1 \end{cases}$$

(10 marks)

8. An engineer is analyzing a Linear Programming problem with the following constraints:

$$x + y \leq 10, x \geq 2, y \geq 3$$

Find the corner points (extreme points) of the feasible region of the above system. *(10 Marks)*

9. In the Simplex Method for a Maximization Linear Programming Problem, why is it impossible to obtain an optimal solution when all the elements in the pivot column are zero or negative? *(10 Marks)*

10. 10. In an ANOVA test conducted for comparing the mean output of four different production processes, the calculated F–value is less than the critical F–value at 5% significance level. State the statistical decision and interpret it in an engineering context. *(10 Marks)*

Part B

Question 02

a. Using the Gauss Divergence Theorem, evaluate $\iint_S \boldsymbol{F} \cdot \boldsymbol{n}\ dS$, where

$$\boldsymbol{F} = (x + y^2)\boldsymbol{i} - 2x\boldsymbol{j} + 2yz\boldsymbol{k},$$

and S is the portion of the plane $x + y + z = 1$ in the first octant.

(Marks 40)

b. Verify Gauss' Divergence Theorem for the vector field

$$\boldsymbol{F} = (x^2 - yz)\boldsymbol{i} + (y^2 - xz)\boldsymbol{j} + (z^2 - xy)\boldsymbol{k}$$

over the rectangular box bounded by $0 \leq x \leq 1$, $0 \leq y \leq 2$ and $0 \leq z \leq 3$.

(Marks 60)

Question 03

a. Verify Stokes' Theorem for the vector field

$$\boldsymbol{F} = (x^2 - y^2)\boldsymbol{i} + 2xy\boldsymbol{j}$$

over a rectangular surface in xy plane bounded by $x = 0, x = a, y = 0$ and $y = a$ where a is a positive real number. *(Marks 60)*

b. Let $\boldsymbol{F} = (\sin y)\boldsymbol{i} + x(1 + \cos y)\boldsymbol{j}$. Evaluate the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise. (Marks 40)

Question 04

a. Find the Laurent series expansions of the function,

$$f(z) = \frac{1}{(z-2)(z-4)} \; ; z \in \mathbb{C}\backslash\{2,4\}$$

at $z_0 = 0$ in each of the following domains.

i. $0 < |z| < 2$. *(10 marks)*

ii. $2 < |z| < 4$. *(10 marks)*

iii. $|z| > 4$. *(10 marks)*

b.

i. Write down the root test for convergence. *(05 marks)*

ii. Find a strip $\{z \in \mathbb{C}: -k < \mathrm{Im}(z) < k\}$ in which the series $\sum_{n=1}^{+\infty} \frac{1}{3^n} \sin nz$ is convergent. *(30 marks)*

iii. What is the largest possible value for k in the strip defined in part (ii)?

(10 marks)

iv. Prove that the series defined in the part (ii) is an analytic function of z in the strip $\{z \in \mathbb{C}: -k < \mathrm{Im}(z) < k\}$. *(25 marks)*

[Hint: Use $|e^{iz}| = e^{-\mathrm{I}\ (z)}$].

Question 05

a. Let $\boldsymbol{a} = (6,0,0)$, $\boldsymbol{b} = (4,k,0)$, $\boldsymbol{c} = (3,0,k)$, $\boldsymbol{d} = (1,1,1)$ be four vectors in the 3-Dimensional cartesian coordinate system and $k \in \mathbb{R}, k \neq 0$.

i. The vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ coplanar if and only if $(\boldsymbol{b} - \boldsymbol{a}) \cdot [(\boldsymbol{c} - \boldsymbol{a}) \times (\boldsymbol{d} - \boldsymbol{a})] = \boldsymbol{0}$. Use this condition to determine the value of k if $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ are coplanar.

(10 marks)

ii. Suppose the coordinate system $OXYZ$ is rotated counterclockwise about the Z −axis, through an angle of $\theta = \frac{\pi}{4}$.

I. Determine coordinates of the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ with respect to the rotated coordinate system. *(25 Marks)*

II. Hence, determine whether the transformed vectors remain coplanar in the rotated coordinate system. Justify your answer.

(20 marks)

b. The moment of inertia tensor $I_{i,j}$ is given by,

$$I_{i,j} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 7 \end{bmatrix}.$$

i. Find eigen values and eigenvector matrix of $I_{i,j}$. Hence construct the orthogonal matrix whose columns are the normalized eigenvectors of $I_{i,j}$.

(25 marks)

ii. Determine the matrix of the tensor components referred to the principal axes of inertia. *(10 marks)*

iii. Verify the result in the above part (ii), by using the Transformation Law of second order tensor, namely $I' = Q^T I Q$, where Q is the orthogonal matrix formed from the normalized eigenvectors. *(10 marks)*

Question 06

a. Use the method of separation of variables to solve the partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{4}\frac{\partial u}{\partial t}.$$

Given that $u(t,0) = u(t,1) = 0$, and $u = \sin \pi x + \sin 2\pi x$ when $t \to 0$.

(Marks 50)

b. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad \text{for} -\infty < x < \infty, t > 0$$

with initial conditions $u(x,0) = 1$ and $\frac{\partial u}{\partial t}(x,0) = 0$, by applying the Fourier Transform.

Assume that $u(x,t) \to 0$ and $\frac{\partial u}{\partial t}(x,t) \to 0$ as $|x| \to \infty$.

[Hint: Take $\int_{-\infty}^{\infty} 1.e^{-i} \quad dx = 2\pi\delta(\omega)$].

(Marks 50)

Question 07

A manufacturing engineer is planning the weekly production of two types of steel support frames used in bridge construction, namely Frame A and Frame B. Each unit of Frame A requires 1 hour of welding and 1 hour of finishing, and each unit of Frame B requires 1 hour of welding and 2 hours of finishing. Due to labor contract agreements, the workshop must utilize at least 120 hours of welding time per week. However, due to equipment limitations, the total finishing time available is restricted to a maximum of 150 hours per week.

The profit contribution from each unit produced is $50 for each unit of Frame A

and $40 for each unit of Frame B. The engineer's objective is to determine the optimal production plan that will maximize the total weekly profit while satisfying the above operational constraints.

a. Define the decision variables. *(10 Marks)*

b. Formulate the Linear Programming Model for the above problem. *(15 Marks)*

c. Convert the Linear Programming Problem into its standard form. *(10 Marks)*

d. Modify the standard form so that it can be solved using the Big M Method.

(10 Marks)

e. Use the Big M Method (Simplex Method) to determine, the optimal number of Frame A units, the optimal number of Frame B units and the maximum weekly profit. *(35 Marks)*

f.

i. Determine whether the welding time constraint is binding or non-binding at the optimal solution. (5 Marks)

ii. If the available welding time is increased from 120 hours to 200 hours, comment (without resolving completely) on how the optimal profit may be affected. (10 Marks)

iii. If the profit of Frame A increases from $50 to $60 per unit, explain whether the current optimal solution is likely to change. Justify your answer.

(5 Marks)

Question 08

A mechanical engineering laboratory is testing the surface roughness produced by three different cutting speeds of a CNC milling machine in order to determine whether the cutting speed has a significant effect on the quality of the finished surface. The surface roughness values (in micrometers) obtained from samples produced under three different cutting speeds are given below:

Cutting Speed A	Cutting Speed B	Cutting Speed C
4.2	5.1	6.0
4.5	5.3	6.4
4.8	5.0	6.2
4.3	5.4	6.5
4.6	5.2	6.1

a. Identify the factor variable and dependent variable. (Marks 10)

b. State the null hypothesis and the alternative hypothesis to test whether the mean surface roughness differs among the three cutting speeds. (Marks 20)

c. Test the hypothesis about the engineers claim at a significant 5% level using ANOVA table and interpret the findings on whether the cutting speed has a significant effect on surface roughness. (Marks 60)

d. State the assumptions you made for this test. (Marks 10)

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Critical Values of the F-Distribution: $\alpha = 0.05$

Denom.	Numerator Degrees of Freedom									
d.f.	1	2	3	4	5	6	7	8	9	10
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	241.882
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165
31	4.160	3.305	2.911	2.679	2.523	2.409	2.323	2.255	2.199	2.153
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.189	2.142
33	4.139	3.285	2.892	2.659	2.503	2.389	2.303	2.235	2.179	2.133
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.170	2.123
35	4.121	3.267	2.874	2.641	2.485	2.372	2.285	2.217	2.161	2.114
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.153	2.106
37	4.105	3.252	2.859	2.626	2.470	2.356	2.270	2.201	2.145	2.098
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.138	2.091
39	4.091	3.238	2.845	2.612	2.456	2.342	2.255	2.187	2.131	2.084
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077
41	4.079	3.226	2.833	2.600	2.443	2.330	2.243	2.174	2.118	2.071
42	4.073	3.220	2.827	2.594	2.438	2.324	2.237	2.168	2.112	2.065
43	4.067	3.214	2.822	2.589	2.432	2.318	2.232	2.163	2.106	2.059
44	4.062	3.209	2.816	2.584	2.427	2.313	2.226	2.157	2.101	2.054
45	4.057	3.204	2.812	2.579	2.422	2.308	2.221	2.152	2.096	2.049
46	4.052	3.200	2.807	2.574	2.417	2.304	2.216	2.147	2.091	2.044
47	4.047	3.195	2.802	2.570	2.413	2.299	2.212	2.143	2.086	2.039
48	4.043	3.191	2.798	2.565	2.409	2.295	2.207	2.138	2.082	2.035
49	4.038	3.187	2.794	2.561	2.404	2.290	2.203	2.134	2.077	2.030
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993
70	3.978	3.128	2.736	2.503	2.346	2.231	2.143	2.074	2.017	1.969
80	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056	1.999	1.951
90	3.947	3.098	2.706	2.473	2.316	2.201	2.113	2.043	1.986	1.938
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.910
140	3.909	3.061	2.669	2.436	2.279	2.164	2.076	2.005	1.947	1.899
180	3.894	3.046	2.655	2.422	2.264	2.149	2.061	1.990	1.932	1.884
200	3.888	3.041	2.650	2.417	2.259	2.144	2.056	1.985	1.927	1.878
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831