0218

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	:	Bachelor of Science Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	:	**MHZ3552 – Engineering Mathematics II**
Academic Year	:	2024/25
Date	:	03rd February 2026
Time	:	0930-1230h
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Eight (8)** pages.
3. This question paper consists of four sections with two questions from each.
4. Please answer a total of **Five (05)** questions choosing **at least one from each single section.**
5. All symbols are in standard notation and state any assumption that you made.
6. All questions carry equal marks.
7. Answer for each question should commence from a new page.
8. This is a Closed Book Test (CBT).
9. Non-programable calculators are allowed.

Part A

Question 01

a. The following data were recorded in a survey of the Wind Turbine Engineering team.
Role in the team: {Aerodynamics, Structural, Electrical, Control Systems} Number of years of turbine design experience. Maximum power output fluctuation (in kW) during 24-hour test cycle. Perception of structural reliability: {Very Poor, Poor, Average, Good, Excellent}

i. Identify the type of data associated with the following variables.

(Marks 06)

A. Perception of structural reliability: .....

B. Maximum power output fluctuation (in kW): .....

C. Number of years of turbine design experience: .....

ii. State the scale of measurement of the following variables.

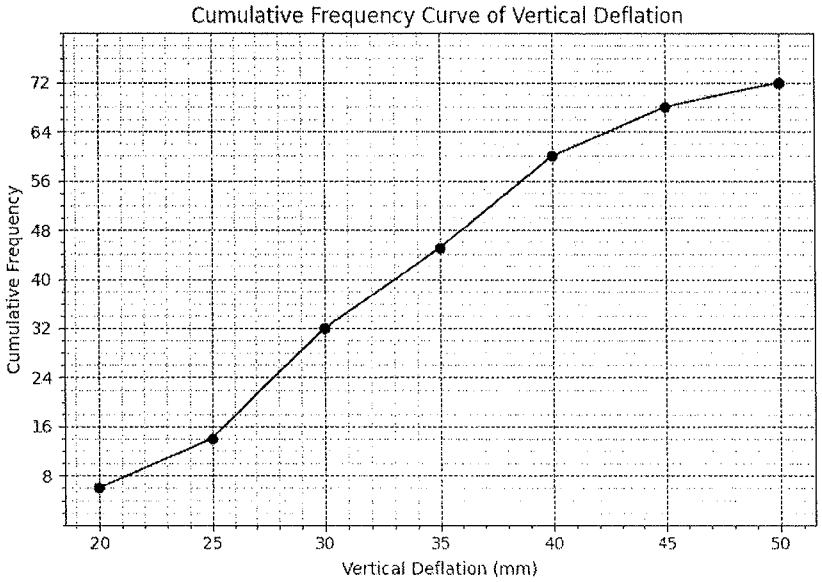
(Marks 06)

A. Perception of structural reliability: .....

B. Maximum power output fluctuation (in kW): .....

C. Role in the team:.....

b. Civil engineer measured the vertical deflection (in mm) of a newly designed bridge deck when a standard truck passes over it at a fixed speed. The maximum deflection is recorded for each test run. The results are summarized in the cumulative frequency graph below.



i. Use the cumulative frequency graph to estimate the median vertical deflection.

(Marks 10)

ii. Considering that the lower quartile is approximately 28.5 mm, use the graph to estimate the upper quartile and hence obtain the interquartile range.

(Marks 10)

iii. The engineer considers any deflection over 42 mm to be unsafe. Determine the percentage of test runs classified as unsafe.

(Marks 10)

c. The following stem-and-leaf diagram shows the thermal conductivity (in $Wm^{-1}K^{-1}$) measured on 21 alumina ceramic insulator samples during quality testing at Thermo Shield Engineering Ltd.

Stem	*Leaf*
0	1 1 2 2 3 4
1	0 1 1 1 2 5 5
2	3 3 4
3	5 6
4	8
5	9
6	9

i. Identify the shape of the thermal conductivity distribution from the diagram.

(Marks 04)

ii. Calculate Mean, Median, Mode. (Marks 20)

iii. Does part (ii) confirm your shape identification in part (i)? Explain.

(Marks 04)

iv. Calculate range and variance. (Marks 20)

v. Recommend the most appropriate central tendency and dispersion measures for this data with reasons. (Marks 10)

Question 02

a. A discrete random variable D represents the number of calibration errors detected during quality testing of pressure sensors in an aircraft system. The probability mass function of D is defined as:

$$P[D = d] = \begin{cases} k(2d+1): d = 0,1 \\ k(8-d) \quad : d = 2,3,4 \\ 0: otherwise \end{cases}$$

where k is a constant.

i. Determine the value of k. (Marks 10)

ii. Find the exact value of $P[D > 1]$. (Marks 10)

iii. Find the expected number of calibration errors $E[D]$. (Marks 10)

b. At Hydro Flow Engineering, the automated welding station fabricates stainless steel pipes for water treatment plants. During a standard 2-hour production cycle, past records show that the average number of weld imperfections is 2.8. If imperfections occur randomly and independently,

i. Determine the rate of weld imperfections per 1-hour production cycle.

(Marks 10)

ii. Compute the probability of no weld imperfections during a 1-hour production cycle. (Marks 10)

iii. Calculate the probability of exactly 2 weld imperfections during a 2-hour production period. (Marks 10)

c. At steel shafts Manufacturing factory, precision steel shafts are produced for automotive engine components. The shafts are supposed to have a diameter of 25.00 *mm*, but actual diameters vary due to machining tolerances. According to production data, shaft diameters follow a normal distribution with a mean of 25.00 *mm* and a standard deviation of 0.15 *mm*.

i. If the acceptable tolerance range is 24.75 *mm* to 25.25 *mm*, what proportion of shafts will meet this requirement?

(Marks 20)

ii. For race car engines require high-RPM performance, only shafts with diameters exceeding 24.92 *mm* qualify for competition specifications. Obtain the proportion of shafts meet this race-grade minimum. (Marks 20)

Part B

Question 03

a. The position vectors of the points A, B, C and D in three – dimensional space are given by

$$\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}, \qquad \boldsymbol{b} = 2\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}, \qquad \boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}, \qquad \boldsymbol{d} = 2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}.$$

i. Find the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$. (Marks 20)

ii. Determine whether the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are linearly independent. Justify your answer. (Marks 20)

iii. Determine whether the points A, B, C and D are coplanar. Give reasons for your conclusion. (Marks 20)

b. The lines L_1 and L_2 pass through pairs of points whose position vectors are given as follows.

For L_1: $\boldsymbol{a_1} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b_1} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$.

For L_2: $\boldsymbol{a_2} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b_2} = 3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

i. Find the vector equations of the lines L_1 and L_2. (Marks 20)

ii. Hence or otherwise, show that the lines L_1 and L_2 intersect. (Marks 20)

Question 04

a. Let $z \in \mathbb{C}$.

i. Using the identity $z + z^{-1} = 2\cos\theta$, express $\cos^4\theta$ in terms of z and z^{-1}.

(Marks 15)

ii. Hence or otherwise, evaluate the integral

$$\int \frac{1}{(1+\tan^2\theta)^2}\, d\theta.$$

(Marks 20)

b. Shade the region of the complex plane represented by the inequality

$$Im\,(z) > |z - 2i|.$$

Clearly indicate the boundary and the region satisfying the inequality on an Argand plane. (Marks 20)

c. Let $f: A \to B$ be a complex - valued function, where $A = \{z_1, z_2, z_3\}$ and $B \subseteq \{z_a, z_b, z_c\}$ and $z_1, z_2, z_3, z_a, z_b, z_c \in \mathbb{C}$. Give diagrammatic examples of such a function f for each of the following cases.

i. f is onto but not one-to-one. (Marks 15)

ii. f is neither one-to-one nor onto. (Marks 15)

iii. f is both one-to -ne and onto. (Marks 15)

Note: clearly indicate the mapping from elements of A to elements of B in each case.

Question 05

a. Consider the following system of liner equations:

$$4x + y + z = 6$$
$$x + 5y + z = 7$$
$$x + y + 6z = 8$$

i. Using the Gauss-Seidel iteration method, perform three iterations starting from $x^{(0)} = y^{(0)} = z^{(0)} = 0$. (Marks 15)

ii. Using the Jacobi iteration method, perform three iterations starting from $x^{(0)} = y^{(0)} = z^{(0)} = 0$. (Marks 20)

iii. Find the exact solution for the system and hence comment on which method is more efficient. (Marks 15)

b. Consider the integral

$$\int_0^6 xe^{x^2} dx.$$

i. Evaluate the integral using the Trapezoidal Rule with six equal subintervals. Give your answer correct to three decimal places.

(Marks 20)

ii. Evaluate the integral using Simpson's Rule with six equal subintervals. Give your answer correct to three decimal places. (Marks 15)

iii. Find the exact value of the integral and hence express its numerical value correct to three decimal places. (Marks 15)

Question 06

a. Obtain the Laplace transform of each of the following functions.

i. $f(t) = e^{-t}\cos 2t$ ii. $f(t) = \mathrm{t}\cos 5t$

(Marks 20)

b. Obtain the Laplace transform of the function $f(t)$ defined by

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ e^t, & t \geq 1 \end{cases}.$$

(Marks 15)

c. Determine the inverse Laplace Transform of

$$F(s) = \frac{1}{(s^3 + 2s^2 + 2s)}.$$

(Marks 25)

d. Solve,

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 1$$

Subject to the initial conditions, $y(0) = 0$ and $y'(0) = 0$. (Marks 40)

Part D

Question 07

a.

i. Define the radius of convergence. (Marks 10)

ii. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n(n^2+1)}.$$

(Marks 15)

b. Let f be a function having derivatives of all orders in an interval containing 0 as an interior point.

i. Write down the McLaurin's series expansion of $f(x)$. (Marks 10)

ii. Write down the McLaurin's series expansion of $f(x) = e^x$. (Marks 25)

iii. Hence deduce the McLaurin's series expansion of $f(x) = \cos x$.

(Marks 25)

iv. Further deduce a closed-form trigonometric expression for $\sum_{n=0}^{\infty} \frac{x^n}{n!} - 2\sum_{n=1}^{\infty} \frac{x^{4n-2}}{(4n-2)!} - 2\sum_{n=1}^{\infty} \frac{x^{4n-1}}{(4n-1)!}$. (Marks 20)

Question 08

a. Show that an even function can have no *sine* terms in its Fourier series expansion.

(Marks 25)

b. A function is defined as,

$$f(x) = x^2 + x \qquad ; -\pi < x < \pi$$

and is extended periodically such that

$f(x) = f(x + 2k\pi)$, where $k \in \mathbb{Z}$.

i. Draw a graph of f in the interval $[-3\pi, 3\pi]$. (Marks 15)

ii. Find the Fourier series expansion of $f(x)$. (Marks 35)

iii. Hence deduce that,

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

(Marks 25)

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