The Open University of Sri Lanka

Faculty of Engineering Technology

Dept. of Mathematics & Philosophy of Engineering

Study Programme	:	Bachelor of Science Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	:	MHZ3551 Engineering Mathematics I
Academic Year	:	2024/25
Date	:	26th January 2026
Time	:	0930–1230h
Duration	:	3 hours
REGISTRATION NO	:	.....

General instructions:

- Read all instructions carefully before answering the questions.
- This paper consists of EIGHT (8) questions in FOUR (4) pages.
- Answer any FIVE (5) questions ONLY. All questions carry equal marks.
- Answer for each question should commence from a new page.
- State any assumptions you required and show all your workings.
- Calculators are not allowed.
- This is a closed book test and do not use red color pen.

Question 01

(a) (i) Translate the following statement into quantifier form.
'If $n \in \mathbb{Z}^+$ and n is not divisible by 2, then the expression $n^2 - 1$ is divisible by 4.'
(Marks 10)

(ii) Use the method of proof by cases to prove the statement in part (i) . *(Marks 40)*

(b) Use the method of indirect proof to prove that 'if $7pq + 5$ is an irrational number, where p and q are real numbers, then p is an irrational number or q is an irrational number'.
(Marks 50)

Question 02

(a) A relation τ is defined on $\mathbb{Z}^+$ by $\tau = \{(x, y)|y = kx \text{ for some } k \in \mathbb{Z}^+\}$.

(i) Determine whether the relation τ is symmetric. Justify your answer. *(Marks 10)*

(ii) Determine whether the relation τ is a partially ordered relation on $\mathbb{Z}^+$. Justify your answer. *(Marks 40)*

(b) Let $f : [2, 3] \to [3, 5]$ be defined by $f(x) = x+2$. Is f an onto function? Justify your answer. *(Marks 10)*

(c) Let $g : \mathbb{R} \to \left(0, \frac{6}{5}\right)$ be defined by $g(x) = \frac{6e^{2x}}{1 + 5e^{2x}}$. Show that the function g is one to one and find the inverse of g. *(Marks 40)*

Question 03

(a) Let $A = \begin{pmatrix} 0 & 0 & 2 \\ 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$. Use Ceyley-Hamilton theorem to find the inverse of matrix A. *(Marks 50)*

(b) Use Gauss elimination to solve the following system of linear equations.

$$\begin{aligned} x + 2y + 3z &= 10 \\ 3x + 7y - 9z &= 14 \\ 2x - 3y - 5z &= -5 \end{aligned}$$ *(Marks 50)*

Question 04

(a) Let $\{x_n\}$ be a sequence defined for each integer $n \geq 1$ such that

$$x_n = \frac{2n+1}{n^2+1} + \frac{2n+1}{n^2+2} + \frac{2n+1}{n^2+3} + \cdots + \frac{2n+1}{n^2+n}.$$

(i) Show that $\frac{2n+1}{n+1} \leq x_n \leq \frac{n(2n+1)}{n^2+1}$ for each $n \geq 1$. *(Marks 30)*

(ii) By using the Sandwich rule find $\lim_{n\to\infty} x_n$. *(Marks 20)*

(b) Show that $\lim_{(x,y)\to(0,0)} \frac{2024x^2y}{2025x^4 + 2026y^2}$ does not exist. *(Marks 20)*

(c) If $f(x, y) = e^{-y}\sin(x + 2y)$, show that $2\frac{\partial f}{\partial x}(x, y) - \frac{\partial f}{\partial y}(x, y) - f(x, y) = 0$. *(Marks 30)*

Question 05

(a) Let f be a function defined on $\mathbb{R}^+$ given by $f(x) = \ln(x+k)$ where $k \in \mathbb{R}^+$.

(i) Using the Mean Value Theorem, show that there exists a point $c \in (a, b)$ such that

$$\frac{(b-a)}{c+k} = \ln(b+k) - \ln(a+k),$$

for any positive real numbers with $a < b$. *(Marks 20)*

(ii) Hence show that for any $a, b \in \mathbb{R}^+$ with $a < b$,

$$\frac{b-a}{b+k} < \ln(b+k) - \ln(a+k) < \frac{b-a}{a+k}. \quad \textit{(Marks 20)}$$

(iii) Hence deduce that $\frac{1}{5} < \ln\left(\frac{5}{4}\right) < \frac{1}{4}$. *(Marks 20)*

(b) Consider the function $f(x) = 1 - x^{\frac{2}{3}}$ on the interval $[-1, 1]$. State whether Rolle's Theorem is applicable to this function on the given interval and justify your answer. *(Marks 40)*

Question 06

(a) Use the substitution $y = vx$ to find the general solution of the following differential equation,

$$x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}$$

subjected to initial condition $y(1) = 0$. *(Marks 50)*

(b) Show that the following differential equation is exact and hence find its general solution.

$$(2xy + 3x^2) + (x^2 + 2y)\frac{dy}{dx} = 0 \quad \textit{(Marks 50)}$$

Question 07

Consider the second-order linear non-homogeneous differential equation,

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 108e^{4x}.$$

(a) Using the D-operator method, find the particular integral, y_p. *(Marks 50)*

(b) Find the complementary solution y_c, and state the general solution. *(Marks 50)*

Question 08

A particle of mass m is projected vertically upwards with an initial velocity u. The motion is resisted by a medium providing a resistive force of mkv^2, where v is the velocity at any time t and k is a constant.

(a) Show that, during the upward motion, the equation of motion is $\frac{dv}{dt} = -(g + kv^2)$.

(Marks 40)

(b) Hence, prove that the time T taken by the particle to reach its maximum height is $T = \frac{1}{\sqrt{gk}} \tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$.

(Marks 60)

End.

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