

THE OPEN UNIVERSITY OF SRI LANKA
 BACHELOR OF SCIENCE DEGREE PROGRAMME – LEVEL 05
 FINAL EXAMINATION - 2025/2026
 PHU 5313 - ADVANCED ELECTROMAGNETISM
 Duration: TWO (02) HOURS



Date 13th January 2026

Time 9.30 am – 11.30 am

$$C = 2.99 \times 10^8 \text{ ms}^{-1}, \epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}, \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

Answer any Four (04) questions only

1. (a) Define the gradient of a scalar field and state its physical significance in the context of electromagnetic theory. Clearly indicate the mathematical operator involved. (3 marks)
 - (b) Write down the del operator in Cartesian coordinates and explain how it operates on
 - (i) a scalar field, and
 - (ii) a vector field.
 Illustrate your explanation by identifying the resulting quantities in each case. (4 marks)
 - (c) Given the vector field $\mathbf{A}(x,y,z) = x^2y\mathbf{i} + yz^2\mathbf{j} + xyz\mathbf{k}$, evaluate the divergence of $\mathbf{A}$ at an arbitrary point (x,y,z) . (5 marks)
 - (d) For a general vector field, discuss the physical meaning of
 - (i) zero divergence, and
 - (ii) zero curl.
 Explain how these conditions are relevant in electrostatics and magnetostatics. (5 marks)
 - (e) A scalar potential field is given by $\phi(x,y,z) = x^2y + yz^3$.
 - (i) Determine the gradient of ϕ .
 - (ii) Hence obtain the corresponding electric field $\mathbf{E}$, given that $\mathbf{E} = -\nabla\phi$.
 - (iii) Evaluate the curl of $\mathbf{E}$ and comment on the result in relation to Maxwell's equations. (8 marks)
2. (a) Write any two vector identities involving the divergence and curl operators. Express each identity in words given by the vector notation. (3 marks)
 - (b) Clearly state the divergence theorem and describe the relationship it establishes between volume and surface integrals. Explain the physical significance of the divergence theorem in electromagnetic theory using an example (4 marks)

- (c) Given a vector field $\mathbf{A}$, explain how Stokes' theorem can be used to relate the circulation of $\mathbf{A}$ around a closed path to a surface integral. Indicate a physical situation in electromagnetism where this theorem is applicable. (5 marks)
- (d) Consider a vector field $\mathbf{B}$ defined in a simply connected region. Discuss how the simultaneous conditions $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} \neq 0$ can arise in electromagnetic systems. Relate your answer to Maxwell's equations in a region with the presence of electric currents or time-varying electric fields. (5 marks)
- (e) The vector field $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ is defined over the volume V bounded by the cube $0 \leq x \leq a, 0 \leq y \leq a, 0 \leq z \leq a$.
- (i) How do you use the divergence theorem to evaluate the $\mathbf{F}$ through the closed surface bounding the cube? (4 marks)
- (ii) Verify the result by directly evaluating the corresponding volume integral. (4 marks)
3. (a) State the uniqueness theorem in electrostatics and explain the boundary conditions under which the electrostatic potential in a region is uniquely determined. (4 marks)
- (b) Explain the method of images and describe why it is useful for solving electrostatic boundary-value problems involving conductors. (4 marks)
- (c) State Green's reciprocity theorem and explain, with an example, how it can be used to relate two electrostatic configurations. (4 marks)
- (d) A point charge q is placed at a distance a above an infinite grounded conducting plane. Calculate the force acting on the charge q using the method of images. (5 marks)
- (e) Using Green's reciprocity theorem, calculate the electrostatic potential at a point P located a distance z above the grounded plane along the vertical line passing through the charge. Show that it matches the potential obtained using the method of images. (8 marks)
4. (a) State Poisson's and Laplace's equations in electrostatics. Clearly distinguish between them. (3 marks)
- (b) Explain Earnshaw's theorem in electrostatics. Discuss its significance for equilibrium positions of charges. (4 marks)
- (c) Briefly describe the general solutions of Laplace's equation and explain the method of separation of variables. (4 marks)
- (d) Consider a rectangular region $0 < x < a, 0 < y < b$, with three sides grounded ($\phi = 0$) and the fourth side at $y = b$ held at potential $\phi_0(x)$. Write the solution of Laplace's equation in two dimensions using separation of variables (without performing the final summation). (4 marks)

(e) For a rectangular region $0 < x < L$, $0 < y < H$, with boundaries,

$$\phi(x, 0) = 0, \phi(x, H) = V_0 \sin(\pi x/L), \phi(0, y) = \phi(L, y) = 0$$

Solve Laplace's equation in two dimensions to find $\phi(x, y)$ explicitly. (10 marks)

5. (a) State the Biot–Savart law and Ampère's circuital law for magnetostatics. Clearly define all symbols used. (4 marks)
- (b) Compare static electric fields and static magnetic fields with respect to their sources, field lines, and work done on charges. (5 marks)
- (c) Write down expressions for:
- (i) the magnetic force on a moving charged particle, and
 - (ii) the magnetic force on a current-carrying conductor element.
- Briefly explain the physical meaning of each expression. (4 marks)
- (d) Explain under what conditions Ampère's law can be used more conveniently than the Biot–Savart law to calculate magnetic fields. Illustrate your answer with one suitable example (no calculation required). (4 marks)
- (e) An infinitely long straight conductor carries a steady current I .
- (i) Using Ampère's law, calculate the magnetic field at a perpendicular distance r from the wire. (4 marks)
 - (ii) A charge q moves with velocity v perpendicular to this magnetic field at that point. Calculate the magnitude of the magnetic force acting on the charge. (4 marks)
6. (a) State the four fundamental laws of electromagnetism that form the basis of Maxwell's equations. (4 marks)
- (b) Starting from the basic laws of electromagnetism, explain briefly how Maxwell modified Ampère's law. Why was this modification necessary? (5 marks)
- (c) Derive Maxwell's equations in differential form from the corresponding integral laws (outline the key steps and state the final equations). (6 marks)
- (d) Explain how Maxwell's equations predict the existence of electromagnetic waves in free space. Clearly state the physical mechanism responsible for wave propagation. (4 marks)
- (e) Using Maxwell's equations in free space, derive the wave equation for the electric field and hence show that electromagnetic waves propagate with speed, $c = \frac{1}{\mu_0 \epsilon_0}$. (6 marks)
