

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme



Department	: Mathematics
Level	: 05
Name of the Examination	: Final Examination
Course Title and - Code	: Linear Programming- ADU5300
Academic Year	: 2025/26
Date	: 05-12-2025
Time	: 09.30 a.m. - 11.30 a.m.
Duration	: Two (02) hours

General Instructions

1. Read all instructions carefully before answering the questions.
 2. This question paper consists of **Six (06)** questions in **Four (04)** pages.
 3. Answer any **Four (04)** questions only. All questions carry equal marks.
 4. Answer for each question should commence from a new page.
 5. Draw fully labeled diagrams where necessary.
 6. Involvement in any activity that is considered as an exam offense will lead to punishment.
 7. Use blue or black ink to answer the questions.
 8. Clearly state your index number in your answer script.
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01. (a) What are the three primary components to be identified in formulating a decision-making problem as an optimization model?
- (b) Let maximum $z = 2x_1 + 3x_2$ be the objective of the linear programming problem subject to the following constraints:

$$2x_1 + x_2 \leq 3$$

$$2x_1 + 3x_2 \geq 6$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

- (i) Solve the above problem graphically.
- (ii) Use the **Big-M method** to reach the optimal solution that you obtained in part (b)(i).

02. Let maximum $z = 3x_1 + 2x_2$ be the objective of the *primal* problem of the linear programming subject to the following constraints:

$$x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \leq 10$$

$$x_2 \leq 3$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

- (a) (i) Find the optimal solution to the above *primal* problem using **graphical method**.
- (ii) Does the removal of the constraint $x_2 \leq 3$ changes the optimal solution to the *primal* problem? Justify your answer.
- (b) Write the *dual* of the above *primal* problem.

Hence, solve the *dual* problem by means of **dual simplex method**.

03. Write down the following linear programming problem in *matrix form*:

$$\text{maximize } z = 6x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_3 \leq 4$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0$$

- (a) Express the *index row formula* and the *formula for the value of z* in matrix form.
- (b) Using the **revised simplex method**, show that the optimal solution to the above problem is $x_1 = 4, x_2 = 6, x_3 = 0$ and $z = 12$.

04. (a) By using calculus, determine the convexity of the two-variable function defined by $f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + x_2^2$.

- (b) Use the **two-phase simplex** method to solve the following linear programming problem:

$$\begin{aligned} &\text{maximize } z = -x_1 + 3x_2 \\ &\text{subject to } \quad x_1 + 2x_2 \geq 2 \\ &\quad \quad \quad 3x_1 + x_2 = 3 \\ &\quad \quad \quad x_1 \geq 0 \text{ and } x_2 \geq 0 \end{aligned}$$

05. A swimming coach forms a team for a 400 meters relay. The relay team consists of four swimmers, swimming four different styles; (I)- back stroke, (II)- breast stroke, (III)- free style and (IV)- butterfly style. Each swimmer must swim 100 meters in one of the four styles. Five swimmers; *A*, *B*, *C*, *D* and *E* are eligible to compete in a relay and the average time taken by each player to complete a 100 meter race is described as follows:

- Swimmer *A* swims the back stroke in 69 seconds, the breast stroke in 75 seconds and has never competed in the free style or butterfly style.
- Swimmer *B* is a free style specialist averaging 61 seconds for the 100 meters, but can also swim the breast stroke in 76 seconds and butterfly style in 80 seconds.
- Swimmer *C* swims all styles; back stroke in 70 seconds, butterfly style in 72 seconds, free style in 65 seconds and breast stroke in 80 seconds.
- Swimmer *D* swims only the butterfly style in 71 seconds.
- Swimmer *E* swims the back stroke in 80 seconds, breast stroke in 76 seconds, free style in 66 seconds and butterfly style in 70 seconds.

- (a) Summarizes the above information in tabular form.
- (b) Formulate a linear programming model to minimize the time of the team to complete the relay.
- (c) Applying the **Hungarian algorithm** to the formulated model in part (b) above, schedule which swimmer should be assigned to which swimming style.

Hence,

- (i) calculate the expected time to complete the 400 meters relay,
- (ii) identify the swimmer who will not be in the relay.

06. A production company has four plants (P_1, P_2, P_3 and P_4) and five warehouses (W_1, W_2, W_3, W_4 and W_5) in different parts of a country. Due to some geographical reasons, a single commodity is sold at different prices in each warehouse. The costs of production and raw materials are also different in each plant due to various factors. The capacities of plants with their relevant costs are summarized in the following table:

Cost per unit	Plant			
	P_1	P_2	P_3	P_4
Production	12	10	8	7
Raw Material	8	7	7	5
Capacity	120	150	110	100

Information on the sales price at each warehouse, transportation costs between warehouses and plants and demand by each warehouse are provided in the table below:

Warehouse	Transportation cost per unit				Sales price per unit	Demand
	P_1	P_2	P_3	P_4		
W_1	4	7	4	3	30	130
W_2	8	9	7	8	32	100
W_3	2	7	6	10	28	120
W_4	10	7	5	8	34	80
W_5	3	5	8	9	30	70

- (a) Derive the *profit table* by using the following formula:
- $$\text{Profit} = \text{Sales price} - \text{Production cost} - \text{Raw material cost} - \text{Transportation cost}$$
- (b) Formulate the problem as a Transportation Problem and find the Initial Basic Feasible Solution (IBFS) to the problem to maximize the profit using **Minimum Cost Method** (MCM).
- Hence,
- show that there exists a degeneracy,
 - determine the profit at the initial solution stage.
- (c) Is the IBFS by means of **Vogel's Approximation Method** (VAM) same as that of from MCM obtained in part (b)? Justify your answer.