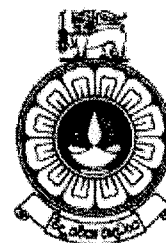


The Open University of Sri Lanka
 B.Sc./ B.Ed. Degree Programme - Level 05
 Final Examination - 2025/2026
 Applied Mathematics
 ADU5307 - Numerical Methods



DURATION: Two Hours

Date: 30.11.2025

Time: 9.30 a.m. - 11.30 a.m.

Answer Four Questions Only.

1. (i) (a) Derive Newton- Raphson formula for solving the equation $f(x) = 0$.
 (b) Show that the equation $x^3 - x - 2 = 0$ has a root in the interval $[1, 2]$ and use the method of the Newton-Raphson with $x_0 = 1.5$ to find a root correct to three decimal places.
 (c) Hence, find all the roots of the equation $x^3 - x - 2 = 0$.
 [Hint: Use Horner's scheme to find polynomial of lower degree]
 (ii) Show that the iterative method can be used to determine the root of $20x = \cos x + 2$ in $(0, 1)$ interval, hence determine a root correct to three decimal points taking $x_0 = 0.8$.

2. (i) State Gregory - Newton backward interpolation formula for $y = f(x)$ for equidistance intervals.
 (ii) The following table shows the concentration $C(x)$ (in mg/l) of a chemical reactant measured at equal time intervals during a laboratory experiment. The readings were taken at every three days over a period of twelve days.

Day (x)	0	3	6	9	12
Concentration $C(x)$ (mg/l)	45	58	76	101	133

Using the Gregory-Newton backward interpolation formula, estimate the concentration of the reactant corresponding to the day 11.

- (iii) Later, it was found that one of the day readings had been recorded incorrectly, and the new data set is shown below.

Day (x)	0	3	7	10	12
Concentration $C(x)$ (mg/l)	45	58	76	101	133

Using the Newton's divided difference formula, estimate the concentration of the reactant corresponding to the day 11.

3. (i) Derive the Lagrange's interpolation formula for $y = f(x)$ for unique intervals.
- (ii) The following data were collected during a controlled laboratory experiment conducted to study the growth of bacteria in a nutrient-rich solution. During the experiment, the number of bacteria (measured in thousands) was recorded at specific time intervals (measured in hours). The recorded observations are summarized in the table below:

Time (hours)	2	5	9
Number of bacteria $N(x)$ (in thousands)	4.8	15.2	40.5

- (a) Find the Second-order polynomial to the above data points by applying Lagrange's interpolation formula.
- (b) Estimate the number of bacteria (in thousands) after 7 hours.
- (c) Find the rate of growth of bacteria at 7 hours.

4. Consider the integral

$$\int_0^{0.3} \ln(1+x) dx.$$

- (i) Find the maximum step size h such that when above function is integrated in the interval $[0, 0.3]$, using the trapezoidal rule, the numerical integrated value is correct to 3 decimal places.
 {Hint: $|E| < \frac{(b-a)h^2}{12}M$; where M is the maximum second derivative of the function in the interval (a, b) }
- (ii) Using the maximum step size h rounded to one decimal place, evaluate the above integral numerically using the Romberg's Method.
 {Hint: Evaluate the integral using the trapezoidal rule with two step sizes, and then apply Romberg's method.}
- (iii) Find the value of this integral by using an analytical method.
- (iv) Hence, find the absolute error of this integral.

5. (i) Consider the following initial value problem

$$y'(t) - \frac{1}{5}y = 2t + 1, \quad y(0) = 1.$$

Find the approximate solution for $y(0.1)$ correct to three decimal places by taking the step size $h = 0.1$ using modified Euler's method.

- (ii) Consider the following initial value problem

$$\frac{dy}{dx} = e^x + \ln(1 + y), \quad y(0) = 0.$$

Find the first two successive approximations for $y(0.1)$ correct to four decimal places using Picard's Method.

- (iii) Applying Taylor series method of third order for the differential equation $y'' = 3y + x^2y'$ subject to the initial conditions $y(0) = 1$, and $y'(0) = -1$, evaluate $y(0.1)$ correct to three decimal places.

6. (i) Consider the following initial value problem

$$\frac{dy}{dx} = x + y, \quad y(0) = 1.$$

Find the approximate solution for $y(0.1)$ and $y(0.2)$ correct to three decimal places using fourth order Runge-Kutta algorithm.

- (ii) Consider the following initial value problem

$$\frac{dy}{dx} = x + y, \quad y(0) = 1, \quad y(0.3) = 1.$$

Find the approximate solution for $y(0.4)$ correct to three decimal places using part (i) two approximation solutions ($y(0.1)$ and $y(0.2)$), by using Milne's predictor-corrector method.

$$[Hint : y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})]$$
