

The Open University of Sri Lanka
 B.Sc/B.Ed. Degree Programme
 Pure Mathematics - Level 05
 PEU5302 Combinatorics
 Final Examination 2025/2026
 Duration: Two hours



Date: 06.12.2025

Time: 9:30 a.m. - 11:30 a.m.

INSTRUCTIONS TO CANDIDATES

- There are **SIX** questions in this paper. Answer **FOUR** of them.
- Calculators are **NOT** allowed.

1. (a) Let $n \in \mathbb{N}$. Find the number of surjections from $\{1, \dots, n\}$ onto $\{1, 2\}$. Hence deduce that the number of surjections from $\{1, \dots, n\}$ onto $\{1, 2, 3\}$ is

$$3^n - 3 \times 2^n + 3.$$

- (b) A mathematics paper consists of Part A and Part B. Part A consists of 4 questions and Part B consists of 6 questions. If a student has to answer 7 questions in total, including at least 3 questions from Part A, how many ways are there to choose the questions to answer the paper?

(100 marks)

2. (a) Four friends Amanda, Bandara, Caldera and Dewmi are going to a restaurant and order 12 paratas. If Amanda wants to eat exactly two paratas, Bandara and Caldera want to eat at least two paratas each and Dewmi wants to eat at least one parata, how many ways are there to distribute the 12 paratas among the friends so that everyone is satisfied?
- (b) Consider a 3×3 matrix with the entries from $\{-1, 0, 1\}$. If the sums of the entries along each of the three rows, three columns, and the two main diagonals are computed, show that at least two of these eight sums are equal.

(100 marks)

3. (a) Show that any postage of Rs.28 or more can be made using only Rs.5 and Rs.8 stamps.
- (b) Draw a flowchart to compute the sum of the squares of the integers from 1 to 100.

(100 marks)

4. You have 4 pure mathematics books, 5 applied mathematics books and 5 novels. You are planning pick 3 books randomly to bring to the university.

- (a) What is the probability that you bring exactly one pure mathematics book?
- (b) What is the probability that you bring at least one pure mathematics book, given that you bring exactly one novel?

(100 marks)

5. (a) Let $n \in \mathbb{N}$ such that $n \geq 3$. Writing the binomial expansion of $(1-x)(1+x)^n$ in two ways, show that for any $2 \leq r \leq n-1$,

$${}^nC_r - {}^nC_{r-1} = {}^{n-1}C_r - {}^{n-1}C_{r-2}.$$

(b) Let $n \in \mathbb{N}$ such that $n \geq 2$. Give a combinatorial argument to show that

$${}^{3n}C_2 = {}^{2n}C_2 + {}^nC_2 + 2n^2.$$

(100 marks)

6. (a) Write the binomial expansion of $\left(2x - \frac{1}{x}\right)^{10}$. Hence find the constant term in the expansion.

(b) Write the multinomial expansion of $(1-x^2+x^3)^{20}$. Hence determine the coefficient of x^{15} in the expansion.

(100 marks)

————— End of the paper —————