

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme



Department	: Mathematics
Level	: 03
Name of the Examination	: Final Examination
Course Title and - Code	: Applied Calculus I – ADE3200
Academic Year	: 2025/26
Date	: 11.01.2026
Time	: 09.30a.m. –11.30a.m.
Duration	: Two Hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of TWO sections: Section A and Section B in Four (04) pages.

Section A

- o This section is **compulsory**.
- o This section carries 100 marks.

Section B

- o This section consists of FIVE(05) Essay Type Questions and each question carries 100 marks.
- o Answer only any THREE(03) questions of them in a separate answer booklet given by the University .

3. Answer for each question should commence from a new page.
4. Involvement in any activity that is considered as an exam offence will lead to punishment.
5. Use blue or black ink to answer the questions.
6. Clearly state your index number in your answer script.

PART A

Answer All Questions

1. a) Determine whether the following statements are **TRUE** or **FALSE**. In each case, justify your answer:

i) If the left-hand and right-hand limits of a function $f(x)$ at $x = a$ exists but are not equal, then the $\lim_{x \rightarrow a} f(x)$ does not exist. (05 marks)

ii) The function f defined by $f(x) = \frac{x^2-9}{x-3}$, is continuous for all $x \in \mathbb{R}$. (05 marks)

iii) If $f''(x) < 0$ on an interval, then the graph is concave up on that interval. (05 marks)

iv) If $\sum \frac{1}{n^2}$ converges, then $\sum \frac{1}{n^3}$ must also converge. (05 marks)

v) If the limit of a sequence does not exist, then the sequence must be unbounded. (10 marks)

vi) If $f(x) = x^x$, then $f'(x) = \frac{1}{1+\ln x}$ is valid only for $x > 0$. (10 marks)

b) Evaluate: $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n-1})$. (10 marks)

c) Find the derivative $\frac{dy}{dx}$ of $x^2 + y^2 = 2xy + 5$ using implicit differentiation. (20 marks)

d) Given the sequence

$$a_1 = 2, \quad a_{n+1} = \frac{1}{2} \left(a_n + \frac{5}{a_n} \right).$$

Find the limit of the sequence $(a_n)_{n \in \mathbb{N}}$, assuming it converges to a finite value (say L) as $n \rightarrow \infty$.

(30 marks)

[Total marks 100]

PART B

Answer only any THREE (03) questions

2. Consider the function f defined by $f(x) = \frac{x^2}{(x+1)(2x-1)}$.

- a) Find the domain of f . (05 marks)
- b) Find x -intercepts and/or y -intercepts of the function, if they exist. (10 marks)
- c) Evaluate $\lim_{x \rightarrow -1^+} f(x)$ and $\lim_{x \rightarrow -1^-} f(x)$. (10 marks)
- d) Find vertical and/or horizontal asymptotes, if they exist. (15 marks)
- e) Determine the intervals on which the function is increasing and the intervals on which the function is decreasing. (20 marks)
- f) Sketch the graph of f . (40 marks)

[Total marks 100]

3. a) Let $f(x) = (1 + x^3)^2$. Show that f is differentiable for all real values of x . (30 marks)
- b) i) Let $f(x) = \ln(1 + x^3)$. Find $f'(x)$ and evaluate $f'(0)$. (10 marks)
- ii) Using $f'(0)$, find the equation of the tangent line drawn to the curve $y = \ln(1 + x^3)$ at the origin. (25 marks)
- c) Let $f(x) = \sqrt{1 + x^4} \cos x^2$. Find $f'(x)$. (35 marks)

[Total marks 100]

4. a) A water tank has the shape of a right circular cone with radius 4m and height 6m, with its vertex pointing downward. The tank is filled with water, and at time $t = 0$ (in minutes), a small hole at the vertex is opened and the water begins to drain out.

When the level of the water in the tank has dropped to 3m from the vertex, the water is flowing out at a rate of $\frac{dV}{dt} = -1.5 \text{ m}^3/\text{min}$. Here, V is the volume of the water at time t .

At that instant, what rate (in meters per minute) is the water level dropping?

(Note: The volume of a cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height) (60 marks)

b) Find k so that

$$f(x) = \begin{cases} kx^2 - 3x, & x < 1 \\ \ln(x+2), & x \geq 1 \end{cases}$$

is continuous at $x = 1$.

Using the definition of continuity, justify your answer.

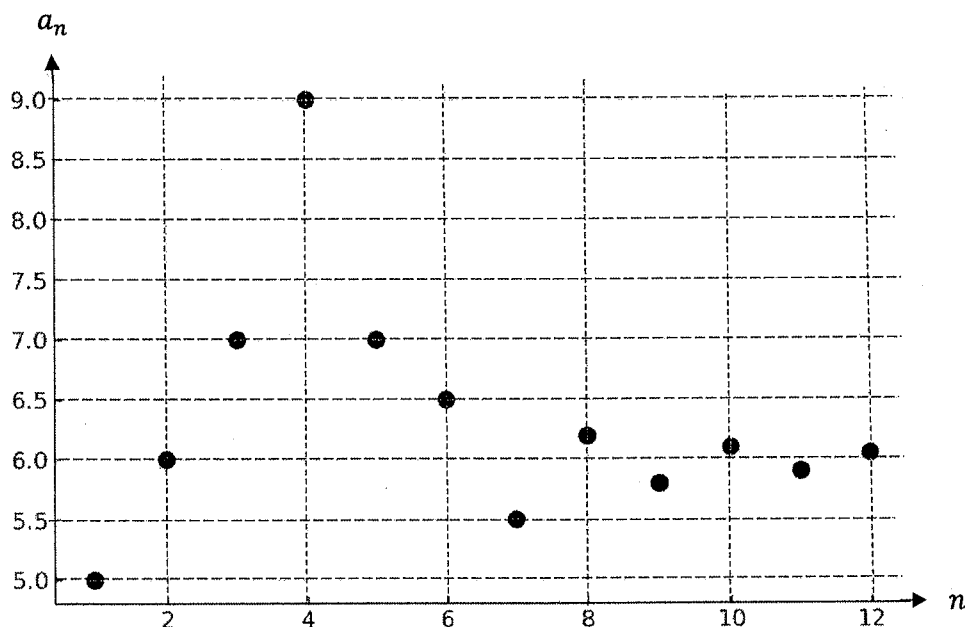
(40 marks)

[Total marks 100]

5. a) State the formal definition of the limit of a sequence a_n as n approaches to infinity.

(05 marks)

b) The following figure illustrates the first 12 points of the sequence a_n .



i) If the sequence approaches a limit as $n \rightarrow \infty$ what is the most likely value of this limit?

(05 marks)

ii) Let L be the value of the limit of the sequence a_n stated in part (i).

Using the formal definition of the limit of a sequence, estimate the smallest integer n_0 such that $|a_n - L| < 0.5$ for all $n > n_0$.

(40 marks)

- d) Consider the sequence with the n^{th} term given by $a_n = \frac{n}{n+2}$. Show that the sequence is **increasing and bounded**.

(50 marks)

[Total marks 100]

6. a) Use **the comparison test** to show that the series $\sum_{n=1}^{\infty} \frac{7n}{3n^3+2}$ is convergent. (50 marks)

- b) Determine whether each of the following series converges or diverges.

i) $\sum_{n=1}^{\infty} \frac{n!}{3^n}$ (15 marks)

ii) $\sum_{n=1}^{\infty} \left(2 + \frac{1}{n}\right)^n$ (15 marks)

iii) $\sum_{n=1}^{\infty} \frac{2^n}{n!n}$ (20 marks)

[Total marks 100]

***** END OF THE QUESTION PAPER *****