

The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Applied Mathematics - Level 03

ADU3300 - Vector Algebra

Final Examination - 2025/2026

Duration: - Two Hours

Date: 02. 12. 2025

Time: 01.30 p.m. - 03.30 p.m.



General Instructions

- This paper consists of **TWO** sections, Section A and Section B. Section A is **compulsory**, it consists of **ONE** Essay Question.
- Section B consists of **FIVE** Essay Questions and answer only **THREE** of them.

SECTION A

Answer all questions.

1. a) Show that the vectors $\underline{u}$, $\underline{v}$, and $\underline{w}$ are linearly independent, where

$$\underline{u} = 2\underline{i} - \underline{j}, \quad \underline{v} = \underline{j} + 3\underline{k}, \quad \text{and} \quad \underline{w} = \underline{i} + \underline{k}.$$

- b) If $\underline{u}$ and $\underline{v}$ are perpendicular unit vectors, show that $|m\underline{u} + n\underline{v}| = \sqrt{m^2 + n^2}$ for any scalars m and n .
- c) Find the equation of the plane passing through the point $A(1, -2, 4)$ and normal to the vector $2\underline{i} - \underline{j} + 3\underline{k}$.
- d) Find the limit of the vector function $\lim_{t \rightarrow 1} \left[\frac{t^3 - 1}{t - 1} \underline{i} + \frac{t^2 - 3t + 2}{t^2 + t - 2} \underline{j} + (t^2 + 1)e^{t-1} \underline{k} \right]$.
- e) Find the Cartesian equation of an ellipse centred at the point with the position vector $4\underline{i} + 2\underline{j}$ and having major and minor axes of lengths 12 and 8 respectively, whose axes are parallel to the coordinate axes.

SECTION B

Answer THREE questions ONLY.

2. a) Given the vectors $\underline{a} = (1, 4, 2)$, $\underline{b} = (3, -1, 5)$, and $\underline{c} = (2, 2, -3)$, verify the vector triple product identity $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c}$.
- b) Using the result $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c}$, where $\underline{a}$, $\underline{b}$ and $\underline{c}$ are given vectors, show that:
- $\underline{i} \times (\underline{a} \times \underline{i}) + \underline{j} \times (\underline{a} \times \underline{j}) + \underline{k} \times (\underline{a} \times \underline{k}) = 2\underline{a}$,
 - $(\underline{a} \times \underline{b}) \cdot (\underline{c} \times \underline{d}) = (\underline{b} \cdot \underline{d})(\underline{a} \cdot \underline{c}) - (\underline{b} \cdot \underline{c})(\underline{a} \cdot \underline{d})$,
 - Deduce that $(\underline{a} \times \underline{b}) \cdot [(\underline{b} \times \underline{c}) \times (\underline{c} \times \underline{a})] = [\underline{a}, \underline{b}, \underline{c}]^2$.
3. a) Find the Cartesian equation of the straight line through the point $A(0, 3, -2)$ and parallel to the line through the points $B(1, 2, 4)$ and $C(-3, 5, 0)$. Verify that the point $D(-4, 6, -6)$ lies on this line.
- b) Let l_1 and l_2 be two straight lines given by $\underline{r} = 2\underline{i} + \underline{j} + 3\underline{k} + \lambda(3\underline{i} - 2\underline{j} + \underline{k})$ and $\underline{r} = 9\underline{i} - \underline{j} + 2\underline{k} + \mu(\underline{i} + 2\underline{j} - 3\underline{k})$ respectively.
- Find the intersection point of the two lines.
 - The point A lies on l_1 when $\lambda = 1$ and the point B lies on l_2 when $\mu = 1$.
Obtain the vector equation of the line l_3 which passes through the points A and B .
4. a) Find the volume of the parallelepiped formed by the vectors,
- $$\underline{a} = (1, 2, 3), \underline{b} = (4, -1, 2), \text{ and } \underline{c} = (0, 5, 1).$$
- b) i) Find the angle between the planes $x + y + z = 6$ and $3x - 17y + 2z = 0$.
- ii) Find the cartesian equation of the plane containing the line $\frac{x-1}{4} = \frac{y+2}{-3} = \frac{z-5}{2}$ and is parallel to the line $\frac{x+2}{6} = \frac{y-1}{-4} = \frac{z+1}{3}$.

5. a) i) A particle moves along a curve, whose parametric equations are

$$x = 3 \cos 3t, y = 3 \sin 3t, z = 2e^{-3t}, \text{ where } t \text{ is the time.}$$

Find the velocity at time t . Hence find the speed and acceleration vector at $t = 0$.

- ii) Let $\underline{a}, \underline{b}$ be constant vectors and ω a constant scalar. For the vector,

$$\underline{r}(t) = \cos \omega t \underline{a} + \sin \omega t \underline{b}, \text{ show that } \underline{r} \times \frac{d\underline{r}}{dt} = \omega \underline{a} \times \underline{b}.$$

- b) A particle moves from the point $(0,0)$ to $(2,8)$ along the parabola $y = 2x^2$. The force field is $\underline{F} = 2y\underline{i} + 3x\underline{j}$. Find the work done by $\underline{F}$ on the particle along this path.

6. a) Given the parametric equations of a circle:

$$\underline{r}(\theta) = (3a + 3a \cos \theta)\underline{i} + 3a \sin \theta \underline{j}$$

- i) Prove that this parametric equation represents a circle of radius $3a$ centered at $(3a, 0)$.

- ii) Determine the coordinates of the point on the circle where $\theta = \frac{\pi}{4}$.

- iii) Verify if the point $(5a, \sqrt{5}a)$ lies on the circle.

- b) Let the space curve be $\underline{r}(t) = 2 \cos t \underline{i} + 2 \sin t \underline{j} + t \underline{k}$.

- i) Find the unit tangent vector.

- ii) Find the principal normal vector and the curvature of the curve.
