The Open University of Sri Lanka

B.Sc/B.Ed Degree programme

Pure Mathematics – Level 03

PEU3300 – Mathematical Logic & Mathematical Proofs

Final Examination - 2025/2026

Date: 16.12.2025	**Time: 01.30 p.m. – 03.30 p.m.**

General Instructions

- **This paper consists of TWO sections, Section A and Section B. Section A is compulsory and it consists of FIVE Structured Essay Questions and carries 100 marks.**
- **Section B consists of FIVE essay type questions and answer only THREE of them.**
- **Each question in Section B carries 100 marks.**
- **This paper consists of 03 pages.**

SECTION A

1. Answer All Questions.

(a) Let p, q, r and s be statements. Find the truth values of the statements p, q, r and s if the statement $\big((p \wedge q) \vee r\big) \Rightarrow (r \vee s)$ is false.

(b) Let $P(x)$: $|x| = 4$ and $Q(x)$: $x = 4$ be the statements defined over the domain $S = \{-4, -3, 1, 5\}$.
Determine all the values $x \in S$, for which the bi-conditional statement $P(x) \Leftrightarrow Q(x)$ is true.

(c) If R is a tautology and S is a contradiction, what can you say about the truth value of $R \Rightarrow S$? Justify your answer.

(d) Write the negation of the statement "There exists a rational number r such that $r \in A$ or $r \in B$."

(e) Prove the statement

$$\exists x \in S, \forall y \in T, Q(x, y)$$

where $Q(x, y)$ means "$x + y$ is prime", $S = \{3,5,7\}$ and $T = \{2,6,8,12\}$.

SECTION B

Answer THREE questions ONLY.

2. (a) Let $x \in \mathbb{Z}$. Consider the statement "$13x - 7$ is even if and only if x is odd".

   (i) Write a logically equivalent statement which is in the form of a conjunctive statement for the above statement.

   (ii) Using the conjunctive statement obtained in part (i), prove the original statement.

   (b) (i) Using the method of proof by contradiction, prove that $1 + \sqrt{2}$ is irrational.

   (ii) Rewrite the above proof as a general statement showing that the sum of a rational number and an irrational number is irrational.

3. Prove or disprove each of the following statements.

   (a) If a, b and c are positive integers then $a^{b^c} = (a^b)^c$.

   (b) There exists an integer whose cube equals its square.

   (c) For any real number r, r^2 is irrational if and only if r is irrational.

   (d) For any even integers a, b, $4|(3a^2 + 7b^2)$.

   (e) There exists a real number y such that for each real number x, $y + x = x$.

4. (a) (i) Show that the product of any two odd integers is an odd integer.

   (ii) Let $a, b \in \mathbb{Z}$. Prove that if ab is odd, then $a^2 + b^2$ is even.

   (b) (i) Below is the proof provided to show that $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

   Fill in the blanks to complete it.

   Assume the contrary ..... is True.

   Then ..... is True and ..... is True.

   $\neg p$ and ..... is ..... Since ..... is True.

   Then both ..... and ..... are False.

   Thus ..... is ..... which is a .....

   Hence $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

(ii) Which technique of proof is being used in part (i)?

5. Prove or disprove each of the following statements.

(a) For each $x \in \mathbb{Z}$, there exists $y \in \mathbb{Z}$ such that $x - y = 2$.

(b) For each $x \in \mathbb{Z}$, for each $y \in \mathbb{Z}$, $x - y = 2$.

(c) There exists $x \in \mathbb{Z}$ such that for each $y \in \mathbb{Z}, x - y = 2$.

(d) For each $x \in \mathbb{N}$, there exists $y \in \mathbb{N}$ such that $x - y = 2$.

6. (a) (i) Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

(ii) Let $x \in \mathbb{Q}$ and $y \in \mathbb{Q}^c$. Prove that there exists $r \in \mathbb{Q}^c$ such that r lies between x and y.

(b) Prove that for every nonnegative integer n, $3|(2^{2n} - 1)$.