



The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Final Examination 2025/2026

Level 04 - Applied Mathematics

ADU4300/ADE4300 Statistical Distribution Theory

Duration: - Two hours

Date: - 03-12-2025

Time: - 9.30 a.m. to 11.30 a.m.

Non programmable calculators are permitted. Statistical tables are provided.

Answer four questions only.

1.

A company that produces a certain electrical product claims that the lifetime of the product X (in years) has the density function.

$$f_X(x) = ke^{-kx}; x > 0, k > 0$$

- (i) Find the cumulative distribution function of the lifetime of the product.
- (ii) Show that the mean lifetime of the product is $\frac{1}{k}$.
- (iii) Show that the variance of the lifetime of the product is $\frac{1}{k^2}$.
- (iv) The past data indicate that it is reasonable to assume that the median of the lifetime is five years. Calculate the value k .
- (v) What is the probability that a randomly selected product will fail within 7 years?

2.

- (a) It is known that the proportion of the pins in a box that are out of the specifications is p . A random sample of n pins was drawn with replacement. Suppose X of them were out of the specifications. The probability mass function of X is given by,

$$P_X(X = x) = nC_x p^x (1 - p)^{n-x}; x = 0, 1, 2, 3, \dots, n$$

Let $M_X(t)$ be the moment generating function of X .

- (i) Show that $M_X(t) = [1 + p(e^t - 1)]^n$

- (ii) Using part (i), show that $E(X) = np$ and $Var(X) = np(1 - p)$
- (b) The number of admissions to an emergency ward of a hospital on a Saturday morning during the period beginning at 12.00 midnight and ending at 2.00 a.m. is found to have an average of 3.5 admissions.

During this period of a particular Saturday morning, find the probability

- (i) that none will be admitted.
- (ii) that two to six persons (inclusive) will be admitted.

3.

- (a) Some traffic lights have three phases: **stop 45%** of the time, **wait or get ready 10%** of the time and **go 45%** of the time. Assuming that you only cross a traffic light when it is in the **go** position and that you have to pass 8 such traffic lights on your way to school.
- (i) Model the number of times that you have to **wait or stop** on your way to school. State any assumptions that must be made and give possible values for the parameters.
- (ii) Find the probability that you have to **wait or stop** only three traffic lights on your way to school.
- (iii) Find the probability that you have to **wait or stop** first time at the third traffic light on your way to school
- (iv) Find the probability that you have to **wait or stop** for the third time at the fifth traffic light on your way to school.
- (b) The time between arrivals of customers at an automatic teller machine is an exponential random variable with a $exp(2)$ distribution. What is the probability that after the arrival of a certain customer, the time until the next three customers is less than 3 minutes?

4.

Suppose that $X_1, X_2, \dots, X_9$ are independent random variables described as

$$X_1 \sim N(3,4) \quad X_2 \sim N(5,6) \quad X_3 \sim exp(3) \quad X_4 \sim gamma(3,3) \quad X_5 \sim exp(3) \quad X_6 \sim Bin(6,0.4) \\ X_7 \sim Bin(4,0.4) \quad X_8 \sim poison(2) \quad X_9 \sim poison(3)$$

Find the following probabilities. Show your calculations and state the justifications clearly.

- (i) $P\left[\left(\frac{X_2 - 2}{3}\right)^2 < 5.024\right]$
- (ii) $P[(X_3 + X_5 + X_4) < 1]$
- (iii) $P[18 < (X_1 + 3X_2) < 25.62]$
- (iv) $P[(X_6 + X_7) > 6]$
- (v) $P[(2X_8 + X_9) > 5]$

5.

A team of 3 is chosen at random from 5 girls and 3 boys. Let X be the number of girls in the team.

- (i) Find the probability mass function of X .
- (ii) Find the probability that the team chosen has more girls than boys.
- (iii) Find the expected number of boys in the team and the variance of the number of boys in the team.
- (iv) Suppose the probability of giving a party by boys when all boys are selected to the team is 0.6, and the probability of giving a party by girls when more girls than boys are selected to the team is 0.8. No party will be given by girls or boys if only two boys are selected to the team. Let Z be the event of giving a party after the selection of the team.
 - a) Find the probability of having a party after the selection of the team.
 - b) Find the joint probability mass function of X and Z .

6.

A particular academic program has three levels and the average marks of *level 1*, *level 2* and *level 3* of a student are given by the variables X_1 , X_2 and X_3 respectively. The *final average* of a student is calculated by the formula.

$$\text{final average} = \frac{X_1 + 2X_2 + 3X_3}{6}.$$

From the past experience it is known that $X_1 \sim N(50, 100)$, $X_2 \sim N(45, 100)$ and $X_3 \sim N(40, 100)$. Assume that X_1 , X_2 and X_3 are mutually independent:

- (i) Calculate the expected *final average* of a student who enrolled in the above academic programme.
- (ii) Calculate the variance of *final average* of a student who enrolled in the above academic programme.
- (iii) Write the distribution of *final average*. Clearly state the values of necessary parameters.
- (iv) Find the proportion of students obtaining *final average* in between 40 and 60.
- (v) Find the lowest final average of the highest 25% of the students.
- (vi) Suppose that 750 students are enrolled to the above programme in 2026 batch. According to the rules and regulations of the above programme, student has to get a minimum of 40 for the *final average*, to complete the programme successfully. Calculate the expected number of students who will complete the course in 2026 batch relying on to the past experience.

Table of $\chi^2_{\alpha, \nu}$ quantiles (χ^2 table)

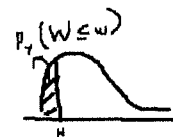
df ν	0.99	0.975	0.95	0.90	α 0.1	0.05	0.025	0.01
1	0	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.02	0.051	0.103	0.211	4.605	5.991	7.378	9.21
3	0.115	0.216	0.352	0.581	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.61	9.236	11.07	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812

Let $X \sim \chi^2_\nu$ and α be a probability. This table contains the upper α quantiles $\chi^2_{\alpha, \nu}$ of the χ^2_ν distributions such that $\Pr(X > \chi^2_{\alpha, \nu}) = \alpha$. For example, $\chi^2_{0.025, 10} = 20.483$.

Left tail values of Standard Gamma Table

W - gamma($\alpha, 1$)

This table contain the probabilities $\Pr(W \leq w)$



w	α					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432

Table of Standard Normal Probabilities



Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(z \leq Z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010