The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 04

Final Examination – 2025/2026

Pure Mathematics

PEU4300 – Real Analysis 1

Duration: - Two Hours.

Date: - 29.11.2025 **Time: - 01.30 p.m.-03.30 p.m.**

Answer Four Questions only.

(01) (a) Using the definition of limit, prove each the following:

(i) $\lim_{n\to\infty} \frac{n^3+2n^2+3n+2}{n^3+n+1} = 1,$

(ii) $\lim_{n\to\infty} \frac{e^n}{e^n-1} = 1.$

(b) Prove that the following sequences are bounded:

(i) $a_n = 1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\cdots+\frac{1}{5^n},$

(ii) $a_n = \frac{5n-1}{30-n^2}$ for each $n \in \mathbb{N}$

(02) (a) Let $x_1 = \sqrt{3}$. Define the rest of the sequence $\langle x_n \rangle$ by $x_{n+1} = \sqrt{3x_n}$ for each $n \in \mathbb{N}$. Prove that,

(i) $\langle x_n \rangle$ is monotonically increasing,

(ii) $\langle x_n \rangle$ is bounded,

(iii) $\langle x_n \rangle$ converges to 3.

(b) Let $x_n = \frac{1}{\sqrt{2n^2+1}} + \frac{1}{\sqrt{2n^2+2}} + \cdots + \frac{1}{\sqrt{2n^2+n}}$. Use Sandwich theorem to prove that $\lim_{n\to\infty} x_n = \frac{1}{\sqrt{2}}$.

(03) (a) Using the definition of a sequence diverges to infinity, prove that the sequence

$\left\langle \frac{2n^2+1}{3n+1} \right\rangle$ diverges to ∞.

(b) Using the definition, show that the sequence $\langle 5 + (-1)^n \rangle$ diverges.

(c) Prove that $\left\langle \frac{5n-2}{7n+4} \right\rangle$ is a Cauchy sequence.

(04) (a) Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+2)(n+4)}$ converges and find the sum.

(b) Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^{\frac{7}{2}}} + \frac{5}{3^{2n-3}} \right)$ converges.

(c) Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^6} + \frac{1}{\sqrt{n}} \right)$ diverges.

(d) Find the sum of $\sum_{n=1}^{\infty} \left(\frac{4}{3^n} + \frac{5}{4^n} - \frac{6}{5^n} \right)$.

(05) Determine the convergence or divergence of each of the following series:

(i) $\sum_{n=1}^{\infty} \left(\sqrt{1+n^8} - n^4 \right)$,

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+2)}$,

(iii) $\sum_{n=1}^{\infty} \frac{n^n e^n}{2n!}$,

(iv) $\sum_{n=1}^{\infty} \left(\frac{3^n+5^n}{7^n} \right)^n$,

(v) $\sum_{n=1}^{\infty} \frac{n^{\frac{5}{6}} + 2n^{\frac{5}{8}} + 4}{n^2 + 3\sqrt{n} - 1}$.

(06) (a) Determine the radius of convergence of each of the following power series:

(i) $\sum_{n=0}^{\infty} \frac{3^n n!}{2n!} x^n$,

(ii) $\sum_{n=0}^{\infty} \frac{1}{5^{2n+1}} x^n$.

(b) Find whether each of the following series is conditionally convergent, absolutely Convergent or divergent:

(i) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$,

(ii) $\sum_{n=1}^{\infty} \frac{-n+6}{n^4+1}$,

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2(n+1)^2}$.