THE OPEN UNIVERSITY OF SRI LANKA

B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04

FINAL EXAMINATION -2025/2026

PURE MATHEMATICS

PEU4302/PEE4302 – Linear Algebra

DURATION: Two (02) Hours

Date: 27.11.2025 **Time:09.30 a.m.-11.30 a.m.**

Answer FOUR questions only.

1. (a) Show that $\begin{vmatrix} 0 & b-a & c-a & b+c \\ a-b & 0 & c-b & c+a \\ a-c & b-c & 0 & a+b \\ b+c & c+a & a+b & 0 \end{vmatrix} = 0$.

(b) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ choose α and β so that $(\alpha I + \beta A)^2 = A$.

(c) Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{pmatrix}.$$

2. (a) Convert the following matrix to the row-reduced echelon form and hence find the rank:

$$A = \begin{pmatrix} 0 & 2 & 4 & 0 \\ 1 & -1 & 0 & 1 \\ 2 & 1 & 2 & 1 \\ 0 & 3 & 2 & -1 \end{pmatrix}.$$

(b) Find non- singular matrices P and Q such that PAQ is normal, where,

$$A=\begin{pmatrix}1 & -1 & 2 & -1\\ 4 & 2 & -1 & 2\\ 2 & 2 & -2 & 0\end{pmatrix}.$$

(c) Let $A=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$. Use Cayley-Hamilton Theorem to express

$A^6-4A^5+8A^4-12A^3+14A^2$ as a linear polynomial in A.

3. (a) Solve the following system of homogeneous equations:

$$x_1-3x_2+3x_3+x_4+x_5=34$$
$$2x_1-6x_2-x_3-2x_4-5x_5=-8$$
$$3x_1-9x_2-5x_3+x_4-11x_5=-20.$$

(b) Determine the values of a and b for which the system

$$x+y-z=0$$
$$2x-y+z=3$$
$$4x+2y+az=b$$

has (i) no solution,

(ii) unique solution,

(iii) infinitely many solutions.

Find the solutions in the cases (ii) and (iii).

4. (a) (i) Prove that A^m has an eigen value λ^m.

(ii) Find the eigen values of the matrix A where:

$$A=\begin{pmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{pmatrix}.$$

(iii) Find the eigen values of A^2 and A^3 using part (i) and (ii).

(b) Transform the quadratic form $-5x_1^2+x_2^2-x_3^2+6x_1x_3+4x_1x_2$ to canonical form by orthogonal transformation. Write the orthogonal transformation which reduces the Quadratic form to Canonical form.

5. (a) Show that $\begin{pmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(b) Solve the following system of equations by LU-decomposition:

$$x + y - 3z + 2w = -5$$
$$2x - y + 2z - 3w = 3$$
$$3x - 2y + z - 4w = 0$$
$$-4x + y - 3z + w = -12.$$

6. (a) Let $A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix}$. Show that A is a unitary matrix.

(b) Show that $B = \begin{pmatrix} 2i & -3 & 4 \\ 3 & 3i & -5 \\ -4 & 5 & 4i \end{pmatrix}$ is skew-Hermitian.

(c) Find the skew-Hermitian form of the matrix in part (b) with $X = \begin{pmatrix} 0 \\ 1 \\ -i \end{pmatrix}$.

(d) Find the eigen values and eigen vectors of the following matrix:

$$\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}.$$