

040

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme : Bachelor of Science Honours in Engineering
Name of the Examination : Final Examination
Course Code and Title : DMX6305 - Modern Control Systems
Academic Year : 2024/25
Date : 3rd March 2026
Time : 0930-1230h
Duration : 3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of five (5) questions on three (3) pages.
3. Answer all questions.
4. All questions carry equal marks.
5. This is a Closed Book Test (CBT).
6. The tables of Laplace transforms, Z-transforms, and important properties and theorems of the Z-transform are attached along with the question paper.
7. The answer to each question should commence on a new page.
8. Answers should be in clear handwriting.
9. Do not use a RED pen.

Question 1

Consider the continuous-time plant $G(s)$ below. The system is sampled using a Zero-Order Hold (ZOH) with a sampling period of $T = 0.2s$.

$$G(s) = \frac{5}{s(s+2)}$$

- a) Write the ZOH equivalent formula below and explain the physical meaning of ZOH in digital control systems briefly. [5 marks]

$$G(z) = \mathcal{Z} \left\{ G(s) \frac{1 - e^{-sT}}{s} \right\}$$

- b) Perform partial fraction expansion of $G(s)$. [5 marks]
c) Apply the Z-transform using standard tables. [5 marks]
d) Derive the discrete-time transfer function $G(z)$. [5 marks]

Question 2

Consider:

$$y[k] - 0.8y[k-1] = u[k] + 0.5u[k-1]$$

Given:

- $y[-1] = 1$
- Input $u[k] = \delta[k]$
- Use unilateral Z-transform

a) Apply the Z-transform, including the initial condition, and determine $Y(z)$.

[10 marks]

b) Obtain the inverse Z-transform of $X(z)$ below using partial fraction expansion.

[10 marks]

$$X(z) = \frac{z}{(z - 0.6)(z - 0.2)}$$

Question 3

Derive the discrete-time state transition matrix A_d for a sampling period $T=0.2s$ for the following continuous-time system with state transition matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -5 & -6 \end{bmatrix}$$

a) Compute the matrix exponential e^{AT} to derive A_d .

[6 marks]

b) Find the eigenvalues of the continuous-time matrix A and comment on the stability of the continuous system.

[7 marks]

c) Find the eigenvalues of the discrete-time matrix A_d and comment on the stability of the discretized system.

[7 marks]

Question 4

A continuous-time unity-feedback control system has the following open-loop transfer function. The system is discretized using the bilinear (Tustin) transformation with a sampling time $T = 0.2$ s.

$$G(s) = \frac{10}{s(s + 2)}$$

- a) Obtain the discrete-time transfer function $G(z)$. [5 marks]
- b) Determine the closed-loop characteristic equation in the z -domain. [5 marks]
- c) Determine whether the discretized feedback system is stable, justifying your answer using z -plane stability concepts. [5 marks]
- d) Briefly discuss how sampling time influences stability in digital feedback systems. [5 marks]

[Hint: Bilinear (Tustin) Transformation, $s = \frac{2z - 1}{Tz + 1}$]

Question 5

A temperature control system for an industrial furnace is modeled by the continuous-time transfer function:

$$G(s) = \frac{15}{s(s + 6)}$$

The system is controlled using a digital controller with:

- Sampling period: $T=0.1$ s
- Zero-Order Hold (ZOH)
- 8-bit ADC and DAC
- ADC input range: ± 20 V
- Actuator saturation limits: ± 15 V
- Unity feedback configuration

Assume:

- The system is causal.
- Zero initial conditions.
- The controller is implemented using fixed-point arithmetic.

- a) Using the ZOH method, write the formula required to obtain the discrete-time transfer function $G(z)$. Without performing a full expansion, explain the steps involved in converting the given $G(s)$ into its discrete equivalent. [5 marks]
- b) Designing a digital proportional controller is $G_c(z) = K$. Write the closed-loop characteristic equation in the z-domain. [5 marks]
- c) For the given 8-bit ADC with a ± 20 V range, calculate the quantization step size. [5 marks]
- d) The actuator has saturation limits of ± 15 V. Explain why actuator saturation is considered a nonlinear element. [5 marks]

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Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Tze^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1} \cos \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1-e^{-aT} z^{-1} \cos \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.	—	—	a^k	$\frac{1}{1-az^{-1}}$
19.	—	—	a^k $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	—	—	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	—	—	$k^2 a^{k-1}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	—	—	$k^3 a^{k-1}$	$\frac{z^{-1}(1+4az^{-1}+a^2 z^{-2})}{(1-az^{-1})^4}$
23.	—	—	$k^4 a^{k-1}$	$\frac{z^{-1}(1+11az^{-1}+11a^2 z^{-2}+a^3 z^{-3})}{(1-az^{-1})^5}$
24.	—	—	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$

$x(kT) = x(k) = 0$ for $k < 0$

Unless otherwise noted, $k = 0, 1, 2, 3, \dots$

Definition of the Z-transform

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t) + bx_2(t)$	$aX_1(z) + bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z) - zx(0)$
4.	$x(t+2T)$	$z^2X(z) - z^2x(0) - zx(T)$
5.	$x(k+2)$	$z^2X(z) - z^2x(0) - zx(1)$
6.	$x(t+kT)$	$z^kX(z) - z^kx(0) - z^{k-1}x(T) - \dots - zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z) - z^kx(0) - z^{k-1}x(1) - \dots - zx(k-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz \frac{d}{dz} X(z)$
11.	$kx(k)$	$-z \frac{d}{dz} X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^a)$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z \frac{d}{dz} X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z \rightarrow \infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z \rightarrow 1} \left[(1-z^{-1})X(z) \right]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k) = x(k) - x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k) = x(k+1) - x(k)$	$(z-1)X(z) - zx(0)$
20.	$\sum_{k=0}^n x(k)$	$\frac{1}{1-z^{-1}} X(z)$
21.	$\frac{\partial}{\partial a} x(t, a)$	$\frac{\partial}{\partial a} X(z, a)$
22.	$k^m x(k)$	$\left(-z \frac{d}{dz} \right)^m X(z)$
23.	$\sum_{k=0}^n x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$