THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY – LEVEL 05
FINAL EXAMINATION – 2004
MPZ 5230/MPZ 5330 – ENGINEERING MATHEMATICS III
DURATION : THREE (03) HOURS

Date : 20th March 2006 **Time:09.30 a.m.-12.30 p.m.**

Answer only Six (06) questions. State any assumptions which are necessary.

01. i. Given the vector $\underline{a} = xyz\underline{i} + (x^2 + y^2 + z^2)\underline{j} - (yz + zx + xy)\underline{k}$,
Verify that
a) div curl $\underline{a} = 0$.
b) curl curl $\underline{a}$ = grad div $\underline{a}$ - $\nabla^2 \underline{a}$

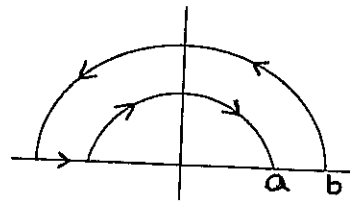
ii. Evaluate the surface integral $\int_s \underline{F}.\underline{n}dA$ Where $\underline{F}$ is the vector filed. $\underline{F} = z\underline{k}$ and S is the curved surface of the hemisphere $x^2+ y^2+z^2 \leq R^2$, $0 \leq z \leq R$.

02. State Gaus theorem. Verity this theorem for the vector field, $\underline{F} = xy\underline{i} + yz\underline{j} + zx\underline{k}$ and the region is defined by, $o \leq x \leq 1$, $o \leq y \leq 1$, $o \leq z \leq 1$.

03. State stoke's theorem and explain its physical meaning.
Verify stoke's theorem for the vector field, $\underline{F} = -y\underline{i} + x\underline{j}$ and the surface S consisting of the upper half-surface of the unit sphere $x^2+y^2+z^2= 1$, $z \geq 0$.

04. i. State Residue Theorem. Evaluate the integral

$$\int_L \frac{z}{z} dz$$



Where L is the contour shown in the figure, by

a) Residue theorem
b) Any other method.

08.

05. In the mapping $W = \dfrac{az+b}{cz+d}$,a,b,c,d are complex constants.

Find the values of those constants so that upper half of the Z-plane is mapped on to the inside of the unit circle.

06. i. a) Explain importance of tensors in engineering.

b) In 3-D space, rank of a tensor is 4. Calculate the number of components of the tensor.

0

ii. The moment of inertia tensor of a certain body is given by, $(I_{ij}) = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ Show that in the coordinate system x_i' defined by $x_i' = \lambda_{ij} x_j$, where $\lambda_{ij} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix}$

$I_{11}' = -2, I_{22}' = 1$ and $I_{33}' = 4$

The transformation for second order tensor is given by,

$T_{st}' = \lambda_{sp} \lambda_{tq} T_{pq}$ in standard notation.

07. Find the normal stresses and directions of principal axes relevant to a stress tensor R, where, $R = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

08. A metal rod AB, length l, is place with A at (0,0) and B at (l,o). The temperature θ satisfies the equation

$$\frac{\partial \theta}{\partial t} = k\frac{\partial^2 \theta}{\partial x^2} \text{ where k>o.}$$

At time t the end A has temperature $\theta_0 e^{-\omega t}$ (ω>o) while B is always kept at zero temperature. Assuming a solution of the type $\theta = Xe^{-\omega t}$, where X is a function of x only, show that the temperature at the mid – point of the rod is $\frac{1}{2}\theta_0 e^{-\omega t} \sec\frac{1}{2}\sqrt{\frac{\omega}{k}}\, l$

09. i. Find the Fourier Integral representations of,

a) $f(t) = e^{-kt}$ when t > 0 and
$f(-t) = f(t)$, with k > 0.

b) $f(t) = e^{-kt}$ when t > 0 and
$f(-t) = -f(t)$, with k > 0

ii. Find the Fourier Transforms of,

a) $f(x)$ = 1 if $-1 \le x \le 1$,
= 0 otherwise

b) $f(x)$ = xe^{-x} if $x \ge 0$,
= 0 otherwise

10. A firm produces three items A, B and C and requires two types of resources man hours and raw material. The following linear programming problems has been formulated to determine the optimum production schedule that maximizes the total profit.

Maximize $Z = 3y_1+y_2+4y_3$

Subject to

$5y_1 + 3y_2 + 5y_3 \leq 45$
$2y_1 + 4y_2 + 5y_3 \leq 30$

Where y_1, y_2. y_3 are the number of items A, B and C.

i. Find the optimal solution using simplex method.
ii. Formulate the dual of the above problem and find its optimal solution.

11. i. Mileage tests were performed to compare three different brands of regular gas. Four different automobiles were used in the experiment, and each brand of gas was used in each car until the mileage was determined. The results, in miles per gallon, are shown in the table.

	Automobile			
Brand	1	2	3	4
A	20.2	18.7	19.7	17.9
B	19.7	19.0	20.3	19.0
C	18.3	18.5	17.9	21.1

a) Is there evidence of a difference in the mean milage rating among the three brands of gasoline? Use $\alpha = 0.05$.

b) Construct the ANOVA summary table for this experiment.

c) Is there evidence of a difference in the mean mileage for the four models? That is, is blocking important in this type of experiment? Use $\alpha = 0.05$.

ii. Describe the steps involved in a simulation process for an engineering project (steps given as a flow chart in preferable)

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