THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
FINAL EXAMINATION - 2005

MEX 5233 –DYNAMICS OF MECHANICAL SYSTEMS

DATE : 04TH MAY 2006
TIME : 1330 HRS - 1630 HRS
DURATION : THREE HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

QUESTION 01

A machine of total mass M is mounted on supports, which provide a total spring stiffness K together with a linear damping force giving a damping ratio β. The machine produces an excitation of the form $P \sin \omega t$, where P is the magnitude and ω is the frequency. Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T = \left[\frac{1+(2\beta r)^2}{(1-r^2)^2+(2\beta r)^2}\right]^{1/2}$$

where $r = \omega/\omega_n$ and $\omega_n = [K/M]^{1/2}$.

Also show that the transmissibility ratio is maximum when

$$r = r_{max} = \frac{1}{2\beta}\left[\sqrt{1+8\beta^2}-1\right]^{1/2}$$

If $\beta = 0.25$, $M = 40$ kg, $\omega = 16$ Hz, what should be the total stiffness of the springs to limit the transmissibility ratio to one tenth of the maximum value ?

You may assume that the linear displacement (x) of the machine at steady state is given by

$$x = \frac{P}{K} Q \sin(\omega t - \phi)$$

where

$$Q = \left[(1-r^2)^2+(2\beta r)^2\right]^{-1/2} \text{ and } \phi = \tan^{-1}\left[2\beta r / (1-r^2)\right]$$

QUESTION 02

A mass m is attached to lower end of each of the two rods of negligible mass and length L and the rods are pivoted about fixed points P and Q as shown in Fig. Q2. Two springs, each of stiffness k, is connected between the rods as shown. When the rods are in the vertical position, the springs are unstretched. Determine the natural frequencies for small oscillations of the system and the corresponding mode shapes.

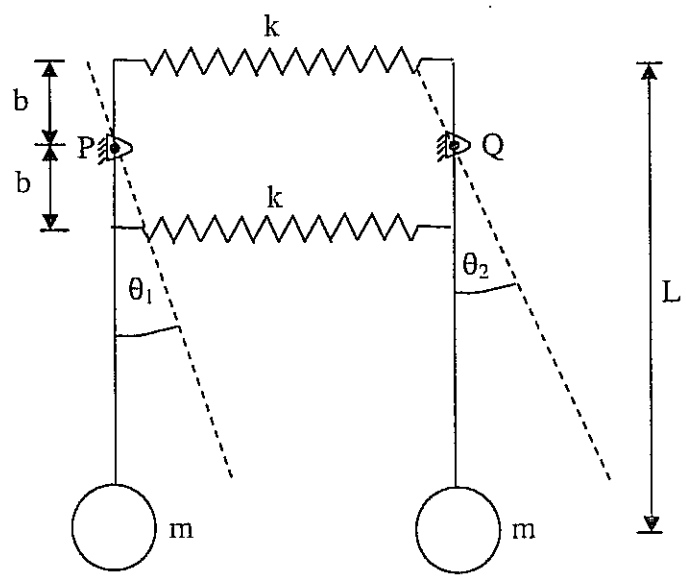


Fig. Q2

QUESTION 03

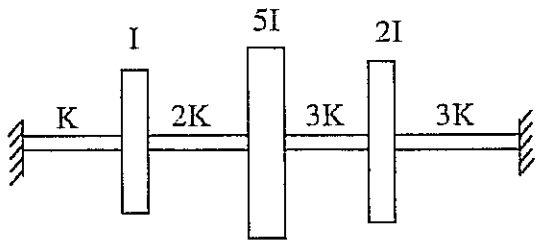


Fig. Q3

Three rotors of moments inertia I, $5I$ and $2I$ are connected as shown in Fig. Q3 by sections of shafts having torsional stiffnesses K, $2K$, $3K$ and $3K$ respectively. Show that the frequency equation can be expressed as

$$[\alpha - 3][\alpha^2 - 4\alpha + 1.3] = 0,$$

where $\alpha = I\omega^2/K$ and ω is the natural frequency of torsional oscillations of the rotor system. Determine the natural frequencies in terms of I and K and the mode shape corresponding to the lowest natural frequency.

QUESTION 04

A uniform beam of mass m per unit length, flexural rigidity EI and length L is fixed at one end and simply supported at the other. Starting from the equation,

$$\frac{\partial^4 y}{\partial x^4} + \frac{m}{EI}\frac{\partial^2 y}{\partial t^2} = 0$$

given in usual notation for the transverse vibration of a beam with constant cross section show that the natural frequencies (ω) of transverse vibration of the beam may be obtained from the roots of the equation,

$$\tanh \lambda L - \tan \lambda L = 0,$$

where $\lambda^4 = m\,\omega^2 / EI$.

QUESTION 05

A uniform beam of flexural rigidity EI and a total mass m over a length L is simply supported at its ends. The dynamic deflection curve is similar to the static deflection curve. Determine the fundamental natural frequency of lateral vibration of the beam using energy method.

Determine the natural frequency of lateral vibration of a beam of I-section, 5 m long, simply supported at the ends and carrying a uniformly distributed load of 800 kg/m. The flexural rigidity of the beam is 28 MNm^2.

QUESTION 06

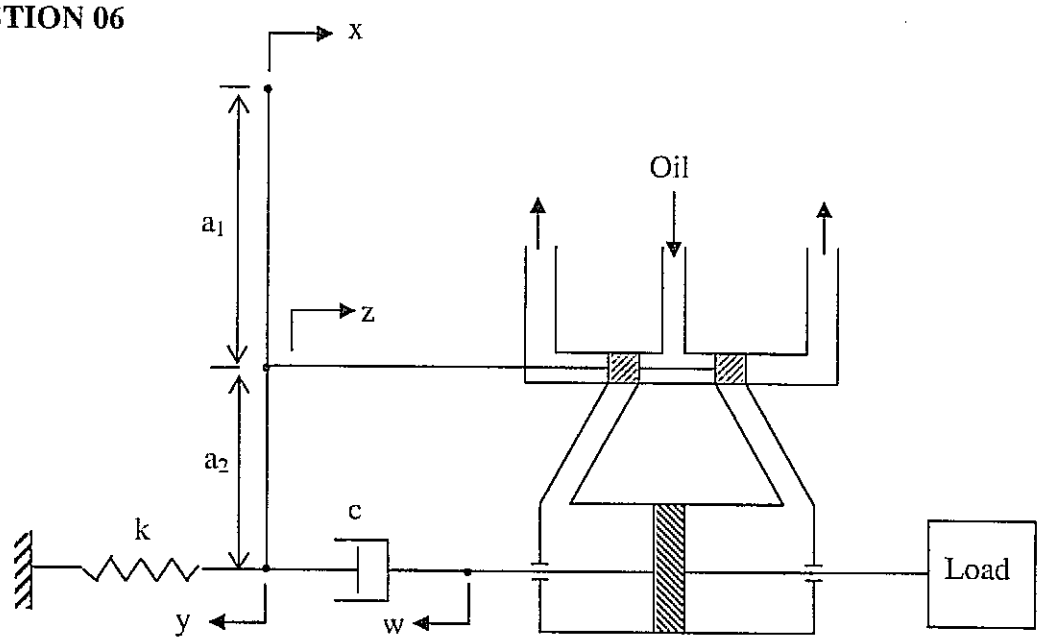


Fig. Q6

The hydraulic servomechanism shown in Fig. Q6 has a 'walking beam', which connects the input position x, the valve position z and the dashpot position y. The piston position is indicated as w. The spring has a stiffness k and the damping constant of the dashpot is c. When z is zero, the valve is fully closed and the walking beam is vertical. The rate of flow of oil to and from the working cylinder, of which the piston has an area of cross section A, is q and it is given by $q = B\,z$, where B is a constant. Draw a complete block diagram of the overall system and obtain the transfer function relating x and w.

QUESTION 07

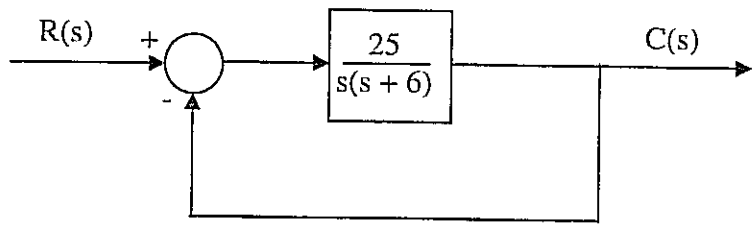


Fig. Q7(a)

Fig. Q7(a) shows a unity feedback control system in which the open loop transfer function is 25/ [s (s + 6)]. Determine the closed loop transfer function of the system and the system response for a unit step function.

Also determine the natural frequency of oscillations, damping ratio, the peak time and the maximum overshoot of the step response.

The damping ratio of the control system is increased by 10%, by incorporating a feed back element as shown in Fig. Q7(b). Determine the value of K and the new maximum overshoot

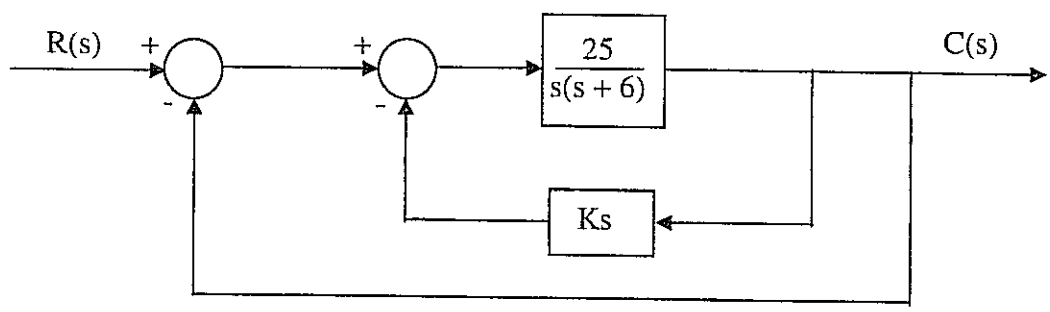


Fig. Q7(b)

QUESTION 08

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{K}{(s+1)(s+2)(s+5)}$$

where K is the gain of the controller. Draw the root locus as a function of K and determine the range of values of K for which the system is stable.

Using the root locus you have drawn show that s = -6 is a closed loop pole of the system when K = 20 and hence derive an expression for the response of the system for a unit impulse input for K = 20. Assume that all initial conditions are zero.

Laplace Transforms

Time function f(t)	Laplace Transform F(S)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n1}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(o)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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