THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
FINAL EXAMINATION - 2006

MEX 5233 –DYNAMICS OF MECHANICAL SYSTEMS

DATE	:	22ND MARCH 2007
TIME	:	1330 HRS - 1630 HRS
DURATION	:	THREE HOURS

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ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

QUESTION 01

Fig. Q1 shows the basic configuration of a vibration measuring instrument consisting of a seismic mass m, a spring of stiffness k and a damper of damping coefficient c. The instrument is placed on a vibrating body having a sinusoidal motion $y = \lambda \sin \omega t$, where λ is the amplitude of vibration and ω is the frequency. The instrument records the relative motion z ($z = x - y$) between the mass and the vibrating body. Show that the amplitude (Z) of the relative motion is given by

$$Z = \frac{\lambda r^2}{\sqrt{\left(1-r^2\right)^2 + 4\beta^2 r^2}}$$

where $r = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$ and $\beta = \frac{c}{2\sqrt{km}}$

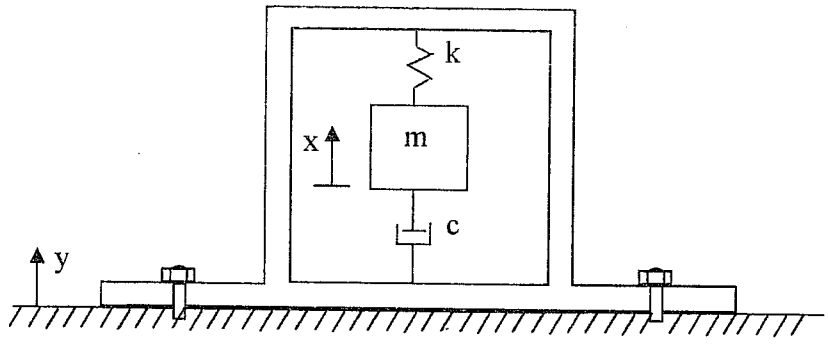


Fig. Q1

Also show that this instrument can be used to measure with sufficient accuracy
(i) the amplitude of the vibrating body if $\omega >> \omega_n$, and
(ii) the acceleration of the vibrating body if $\omega << \omega_n$.

You may use the solution of the second order differential equation given in Appendix.

QUESTION 02

Fig. Q2 shows a compound pulley which can rotate about a fixed axis through O. A light string connects the spring of stiffness k_3 to the larger radius r_2 of the compound pulley. One end of the spring of stiffness k_2 is connected to the smaller radius r_1 of the pulley and the other carries a mass m which is supported by another spring of stiffness k_1. The moment inertia of the compound pulley about the fixed axis through O is I.

Determine the equations of motion and the frequency equation for small angular oscillations.

Show that if $r_1 = r$, $r_2 = 2r$, $k_1 = k_2 = k_3 = k$ and $I = 5mr^2$, the frequency equation reduces to

$$\omega^4 - 3\omega^2\alpha + 1.8\alpha^2 = 0$$

where $\alpha = k/m$ and ω is the natural frequency of oscillations.

Also determine the natural frequencies in terms of k and m.

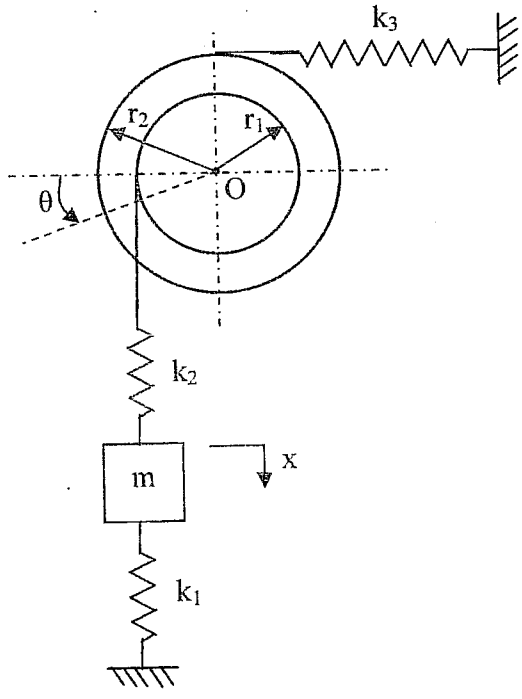


Fig. Q2

QUESTION 03

Fig. Q3 shows the arrangement of rotors R_1 & R_2 and gears G_1 & G_2. The rotors and gears are connected by shafts S_1, S_2 and S_3 as shown. The number of teeth on gear G_2 is two times that on gear G_1. The rotors and the gears have the parameters given overleaf.

Moment of inertia : Rotor $R_1 - 2I$ kgm^2
Rotor $R_2 - 2I$ kgm^2
Gear $G_1 - I$ kgm^2
Gear $G_2 - 6I$ kgm^2

Torsional stiffness : Shaft S_1 = Shaft S_2 = Shaft S_3 = K

Working from fundamentals, show that the frequency equation for natural frequencies of torsional oscillations of the system can be expressed as

$$(\alpha - 0.5)(\alpha^2 - 1.5\alpha + 0.05) = 0$$

where $\alpha = \dfrac{I\omega^2}{K}$ and ω is the natural frequency of torsional oscillations.

Determine the natural frequencies in terms of K and I.

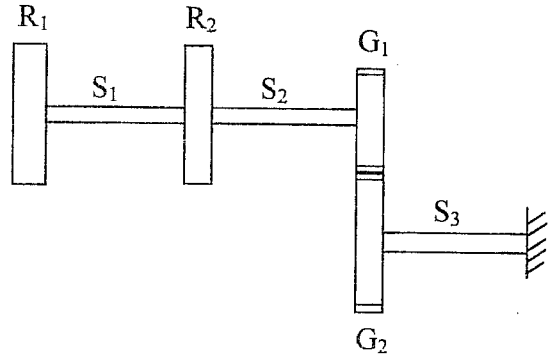


Fig. Q3

QUESTION 04

A uniform beam of mass m per unit length, flexural rigidity EI and length L is pinned at one end and free at the other. A transverse harmonic force, $P\sin \omega t$, where P is the magnitude and ω is the frequency, is acting at the free end. Starting from the equation, $\dfrac{\partial^4 y}{\partial x^4} + \dfrac{m}{EI}\dfrac{\partial^2 y}{\partial t^2} = 0$,

given in usual notations, for the transverse vibration of a beam with a constant cross section, show that the amplitude (U) of the forced vibrations under the load can be expressed as,

$$U = \frac{2P\sin \lambda L \sinh \lambda L}{EI\lambda^3[\cos \lambda L \sinh \lambda L - \sin \lambda L \cosh \lambda L]}$$

where $\lambda^4 = m\,\omega^2 / EI$.

QUESTION 05

A uniform beam of flexural rigidity EI and a total mass of M over a length L is fixed at each end and carries a point mass $2M$ at the centre. The dynamic deflection curve is of the form $y = a\sin^2\left(\frac{\pi x}{L}\right)$, where a is a constant and y is the deflection of the beam at a distance x from one end. Determine the fundamental natural frequency of lateral vibration of the beam using the energy method.

A uniform beam with a circular cross section is fixed at each end. It has a length of 4 m and mass 60 kg and carries a point mass equal to twice its mass at the centre. What should be the diameter of the beam if the fundamental frequency of lateral vibration of the beam is to be 5 Hz. Modulus of elasticity of the material of the beam is 200 GN/m^2.

QUESTION 06

Fig. Q6 shows a simplified model of the suspension system of a vehicle in which the mass of the vehicle m is supported by a spring of stiffness k and a dashpot with a damping constant c. The motion x_i at point P is the input to the system and the motion x_o is the output. Derive the differential equation governing the motion of the system and determine the transfer function relating $X_o(s)$ to $X_i(s)$. Assume that the motion of the body is confined to the vertical direction only.

If m = 1000 kg, find k and c so that the system is critically damped with an undamped natural frequency of 10 rad/s and hence determine the response $x_o(t)$ for a unit impulse input of $x_i(t)$. Assume zero initial conditions.

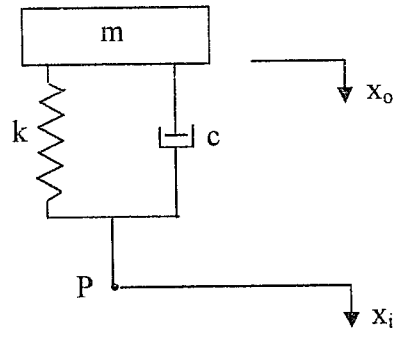


Fig. Q6

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{25}{s(s+2)}.$$

Determine the damping ratio, undamped natural frequency and damped natural frequency of this control system.

To improve the dynamic performance of the above control system, a tachometer feed back is added. Determine the value of the tachometer constant such that the resulting system will have a damping ratio of 0.5. Also determine the response of this system to a unit step input. Assume that all initial conditions are zero.

QUESTION 08

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{K(s+2)}{s(s+3)(s^2+4s+5)}$$

where K is the gain of the controller. Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

Also determine the gain of the controller that makes the relative damping of the overall system equals to 0.707.

APPENDIX

1. Solution of a Second Order Differential Equation.

The particular integral of a differential equation of the form, $\ddot{x} + 2\beta\omega_n\dot{x} + \omega_n^2 x = P\sin\omega t$, is given as,

$$x_P = \frac{P\sin(\omega t - \phi)}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\beta\omega\omega_n\right)^2}} \quad \text{where } \phi = \tan^{-1}\left(\frac{2\beta\omega\omega_n}{\omega_n^2 - \omega^2}\right)$$

2. Laplace Transforms

Time function f(t)	Laplace Transform F(S)
Unit Impulse δ(t)	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	sF(s)-f(o)
$\frac{df^n(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0).....f^{n-1}(0)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
sin ωt	$\frac{\omega}{s^2+\omega^2}$
cos ωt	$\frac{s}{s^2+\omega^2}$
e^{-at} sin ωt	$\frac{\omega}{(s+a)^2+\omega^2}$
e^{-at} cos ωt	$\frac{s+a}{(s+a)^2+\omega^2}$

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