THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
FINAL EXAMINATION – 2007

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MEX5233 – DYNAMICS OF MECHANICAL SYSTEMS

DATE : 10 TH MAY 2008
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

A machine of mass m is mounted on the ground through an isolator consisting of a spring of stiffness k and a dashpot having a damping coefficient c. The machine produces an excitation of the form P sin ωt, where P is the magnitude and ω is the frequency. Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T = \left[\frac{1 - (2\beta r)^2}{(1-r^2)^2 + (2\beta r)^2}\right]^{1/2}$$

where $r = \dfrac{\omega}{\omega_n}$, $\beta = \dfrac{c}{2m\omega_n}$ and $\omega_n = \left(\dfrac{k}{m}\right)^{1/2}$.

The following observations were made with respect to performance of the above machine of mass 200 kg.

(i) Transmissibility at resonance was equal to 2.

(ii) Transmissibility at an excitation frequency of 20 Hz was equal to 0.5.

Calculate the damping constant of the dash pot and the stiffness of the spring.

You may assume that the linear displacement (y) of the machine at steady state is given by

$$y = \frac{P}{k} Q \sin(\omega t - \phi), \text{ where } Q = \left[(1-r^2)^2 + (2\beta r)^2\right]^{-1/2} \text{ and } \phi = \tan^{-1}\left[\frac{2\beta r}{(1-r^2)}\right].$$

QUESTION 02

One type of Seismograph (a device that records earthquakes) can be modeled as shown in Fig. Q2. It consists of a mass M, a spring of stiffness k, a rod of negligible mass and a mass m attached to one end of the rod, the other end of which is pivoted..

(i) Determine the equations of motion of the system using Newton's law. Verify your result by Lagrange's method.

(ii) Determine the frequency equation.

(iii) Evaluate the frequencies if $a = 0.5$ m, $L = 1$ m, $m = 0.5$ kg, $M = 1$ kg, $k = 200$ N/m and $g = 10$ m/s^2.

(iv) Determine the modes.

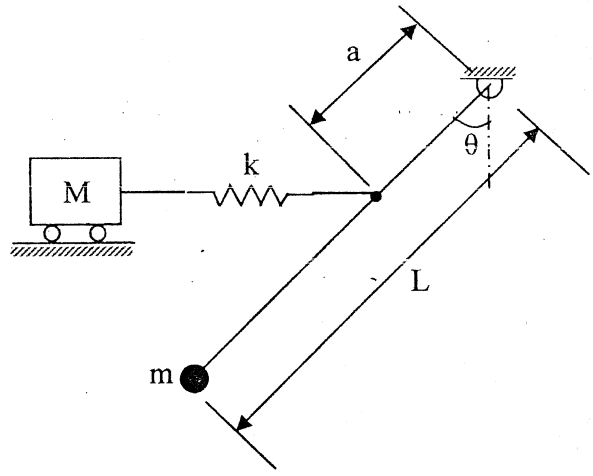


Fig. Q2

QUESTION 03

Fig. Q3 shows a system of rotors. The moments of inertia of the rotors and the torsional stiffnesses of the shaft sections are as indicated in the figure. Obtain the frequency equation of the system.

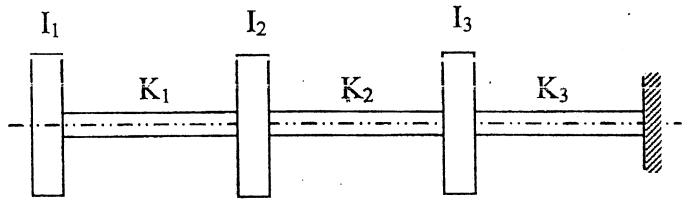


Fig. Q3

If $I_1 = I$, $I_2 = 2I$, $I_3 = 4I$, $K_1 = K_2 = K$ and $K_3 = 3K$, show that the frequency equation can be reduced to

$$(\alpha - 1)(8\alpha^2 - 16\alpha + 3) = 0,$$

where $\alpha = \dfrac{I\omega^2}{K}$ and ω is natural frequency of torsional vibration.

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Determine the natural frequencies in terms of K and I and the corresponding mode shapes.

QUESTION 04

Determine the natural frequency of transverse vibration of a uniform beam with fixed ends and of length L, mass per unit length m and flexural rigidity EI using Rayleigh's method. Assume that the dynamic deflection curve of the beam is of the form,

$$y = \lambda[x^2(L - x)^2] \ ,$$

where y is the deflection of the beam at a section at a distance x from one end. L is the length of the beam and λ is a constant.

A thin walled steel pipe with a bore diameter of 70 mm has a mass of 9.25 kg/m and a flexural rigidity of 0.349 MNm^2. It is held in clamped supports 4 m apart, and is full of liquid of density 900 kg/m^3. Calculate the natural frequency of transverse vibration of the pipe.

If the bore diameter of the pipe is increased to 80 mm, keeping the same wall thickness what would be the new frequency of lateral vibration.

QUESTION 05

A mechanism of a pressure measuring instrument is shown in Fig. Q5. In the figure AE is a rigid uniform lever of mass m and length $4a$ and it is pivoted at A. Pressure p applied at the piston of the cylinder is measured by the displacement x of end E of lever. The cross sectional area of the piston is b. The mass of the piston and the piston rod are negligible. The spring in the cylinder has a stiffness k_1 while the other spring attached to the rod at D has a stiffness of k_2. The dashpot connected to the rod at C has a damping coefficient c. Obtain a transfer function relating the displacement (x) and (p).

If $m = \dfrac{1}{16}$ kg, $c = \dfrac{1}{6}$ Ns/m, $k_1 = 21$ N/m, $k_2 = 1$ N/m and $b = 1 \times 10^{-3} m^2$, determine the response of the system for a step input of pressure 1 kN/m^2. Also obtain the settling time using two percent criteria.

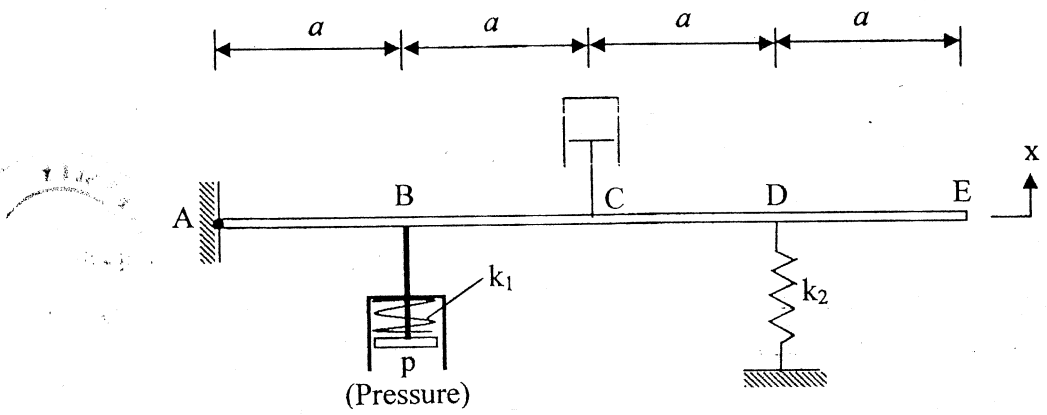


Fig. Q5

QUESTION 06

The block diagram of a position control system with velocity feed back is shown in Fig. Q6. If the transient response of the closed loop system has a damping ratio of 0.6, and a frequency of damped oscillation of 12 rad/s,

(i) determine the values of K_1 and K_2,

(ii) determine the steady state position error due to a velocity input signal of 1 rad/s, and

(iii) derive an expression for the output transient response due to a unit step input function.

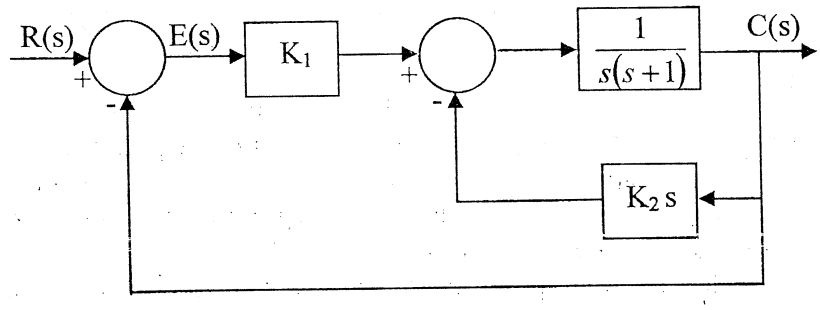


Fig. Q6

QUESTION 07

The open loop transfer function of a unity feedback control system is given by

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$$G(s) = \frac{K}{(s+2)(s^2+8s+20)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(ii) Using the root locus you have drawn show that s = -6 is a closed loop pole of the system when $K = 32$. Determine the other closed loop poles.

(iii) Calculate the damping ratio of the closed loop system when $K = 32$.

QUESTION 08

Define the gain margin and the phase margin of a unity feed back control system with respect to a Nyquist plot.

The open loop transfer function of a unity feed back control system is given by

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Determine the value of K in order to have a

(i) gain margin of 12 db

(ii) phase margin of 30°.

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(o)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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