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BACHELOR OF TECHNOLOGY – LEVEL 05

FINAL EXAMINATION – 2007

MPZ 5230/MPZ 5330 – ENGINEERING MATHEMATICS III

252

DURATION : THREE (03) HOURS

Date : 04th May 2008 **Time: 0930 – 1230 hrs.**

Instructions

- **Answer only Six (06) questions.**
- **No. of pages in the paper - 04**
- **State any assumptions you use.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**
- **Request statistical table.**

01. a) Let R be the region of the parallelogram bounded by the lines $x + y = 1$, $x + y = 2$, and $2x - 3y = 2$, $2x - 3y = 5$.

Substitute $u = x + y$, $v = 2x - 3y$ to find its area

$$A = \iint_R dxdy$$

b) Show that,

$$\iint_A \sqrt{4a^2 - x^2 - y^2}\, dxdy = \frac{4}{9}(3\pi - 4)a^3,$$ Where A is the region within the semicircle $x^2 + y^2 - 2ax = 0$ lying in the upper half plane.

02. a) In usual notation, Prove that for a vector field $\underline{F}$ and scalarfield ϕ.

i. $\nabla . \phi\underline{F} = \phi \nabla . \underline{F} + \underline{F} . \nabla \phi$ and

ii. $\nabla \times (\phi\underline{F}) = \nabla\phi \times \underline{F} + \phi \nabla \times \underline{F}$

b) Prove that $r^n\underline{r}$ is irrotational, where n is an integer, find n when it is solenoidal.

03. a) State Stokes Theorem, and using Stoke's Theorem deduce greens' Theorem in plane.

b) Show that the area bounded by a simple closed curve C in the xy plane is given by

$$\frac{1}{2}\oint_C(-ydx + xdy)$$

c) Under the coordinate transformations, x = rcos θ, y = rsin θ, show that $x\,dy - ydx = r^2 d\theta$. Hence evaluate the area bounded by both loops of the leminicate $r^2 = a^2 \cos\theta$.

04. a) i. State Gauss divergence theorem.

ii. If $\underline{F} = x^3\,\underline{i} + y^3\,\underline{j} + z^3\underline{k}$ and S is a sphere of radius a, bounding a volume V. Show by means of divergence theorem, that $\int_S \underline{F} \cdot \underline{n}\,ds = \int_V 3(x^2 + y^2 + z^2)dv$

where $\underline{n} = \frac{1}{a}(x\underline{i} + y\underline{j} + z\underline{k})$

b) Transforming in to spherical polar coordinates, Show that the value of surface intergral is $\frac{12\pi a^5}{5}$.

05. a) Find the Taylor series expansion of the following function about z = a

$$f(z) = \frac{2-3z}{2z^2 - 3z + 1},\ a = -1$$

b) Find the Laurent series of the following function about the indicated singularity.

$$f(z) = \frac{z}{(z+1)(z+3)} \text{ at } z = -2$$

c) Find the bilinear transformation, which sends the points z = -1, 0, 1 in to the points w = 0, i, 3i respectively.

06. a) State Residue's Theorem.

b) Show that $\int_0^{2\pi} \frac{\cos 3\theta}{(5 - 4\cos\theta)} d\theta = \frac{\pi}{12}$

(Hint : Let $Z = e^{i\theta}$, $0 \le \theta \le 2\pi$)

07. a) Define Fourier Transformation of the function f(x).

b) Show that

i. $F_S(xf(x)) = -\frac{d}{ds}F_c(S)$ Where $F_c(S)$ = Fourier cosine transform of f(x)

ii. $F_c(xf(x)) = \frac{d}{ds}F_S(S)$ Where $F_S(S)$ = Fourier sine transform of f(x)

Hence find fourier cosine and sine transform of xe^{-ax}

08. The components of a moment of inertia tensor are given by

$$[I_{ij}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

a) Determine the eigen values and the principal axes of the tensor.

b) Express the tensor with respect to the principal axes.

c) Write the equation of the transformation relating the co-ordinates X_{ij} with that determined by principal axes.

09. a) State transformation Law of Vectors.

b) Write down whether scalars are tensors or not

c) If your answer for above question is 'Yes, scalars are tensors' write down the rank of scalar.

d) Let φ be scalar in x_i system. Given a transformation of coordinates $x_i' = \lambda_{ij} x_j$

Where

$$\lambda_{ij} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{3}}{3} & \frac{-\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{-\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{2\sqrt{6}}{6} \end{bmatrix}$$ Find the value of φ in x_i' system.

10. Suppose that in an agriculture experiment two different chemical treatment of soil were conducted. The numbers of bushels per acre were measured and are shown below. Is there a significant difference in these two chemicals? Use 5% significance level.

Number of bushels per acre

Field Number	Chemical X	Chemical Y
1	17.5	14.6
2	16.9	16.7
3	15.8	20.8
4	18.6	18.9
5	16.4	17.5
6	19.2	19.2
7	17.7	16.5
8	15.4	20.5
9	20.3	18.3
10	15.7	16.2
11	17.8	17.5
12	18.9	29.1

11. a) Define each of the following terms with respect to a linear programming problem

i. Slack variable ii. Surplus Variable

iii. Infeasible solution iv. Feasible solution

v. Artificial Variable

b) A manufacturing firm produces four products x_1, x_2, x_3 and x_4. Each product requires material and machine processing. The linear programming model is formulated as

Maximize $Z = 2x_1 + 8x_2 + 10x_3 + 6x_4$

Subject to

$2x_1 + x_2 + 4x_3 + 2x_4 \leq 200$ (material, lbs)

$x_1 + 2x_2 + 2x_3 + x_4 \leq 160$ (machine processing, hr)

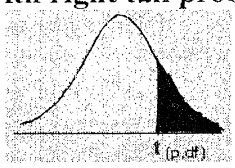
$x_1, x_2, x_3, x_4 \geq 0$

Find the optimal solution using **simplex method**.

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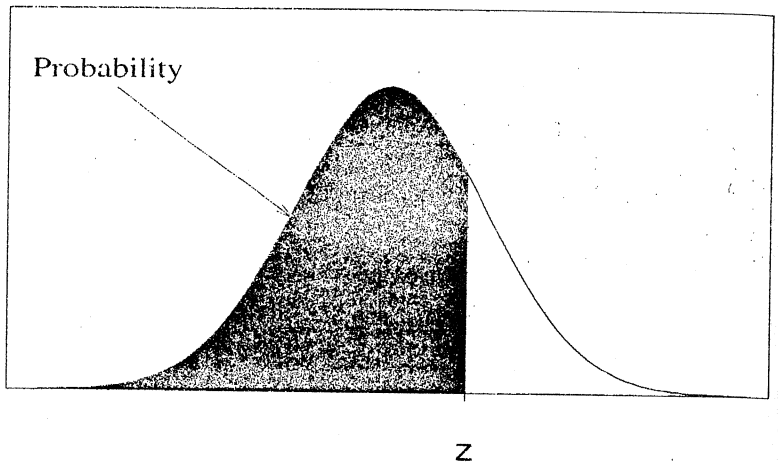
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t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Table entry for z is the probability lying to the left of z



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998