THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 06

FINAL EXAMINATION – 2007

MPZ 6231 – DISCRETE MATHEMATICS (ESSAY TYPE PAPER)

036

DURATION : THREE (03) HOURS

Index No.....

Date : 11th May 2008 Time: 0930 - 1230 hrs.

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Instructions:

- **Answer only six questions**
- **State any assumption you required**
- **Do not spend more than 30 minutes for one problem**
- **Show all your workings**
- **All symbols are in standard notation.**

01. a) What do you mean by "two propositional statements P &Q are logically equivalent"?

b) Let p, q, r and s be four propositions. Let P denotes the propositional statement $s \rightarrow (p \wedge r) \wedge [(p \rightarrow (r \vee q) \wedge s)]$ and Q denotes the propositional statement $(p \vee r)$.

i. Draw a truth table to calculate the truth values of P and Q.

ii. State whether the P and Q are logically equivalent. Justify your answer.

iii. Using the above truth table show that the propositional statement $[(p \rightarrow (r \vee q)) \vee (p \vee r)]$ is a tautology.

02. a) i. Write down the contrapositive of the statement "$p \Rightarrow q$" where p and q are propositions.

ii. Let n be an integer. Prove that if n^2 is odd, then n is odd.

b) i. Using mathematical Induction prove that

$1 + 2 + ... + n = \frac{n}{2}(n+1)$ for every positive integer n.

ii. Let $x \in \Re$ with $x > -1$ and $x \neq 0$. Show that $(1 + x)^n > 1 + nx$ for every positive integer $n \geq 2$.

03. a) Without using Venn diagram show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ for sets A, B and C.

b) Let A,B,C are finite sets. Assume that number of elements in the set $A \cup B$ is given by the formula $|A \cup B| = |A| + |B| - |A \cap B|$ obtain the similar formula for $|A \cup B \cup C|$

c) i. Define $X \oplus Y$ and $X \setminus Y$ for sets X and Y.

ii. Let V = {1, 2, 3, 4}. Find subsets A, B, C of V such that $A \oplus B \setminus C \neq A \oplus (B \setminus C)$

04. a) Let A = {1,2,3}

i. Find $A \times A$ and verify that R is a relation from A to A, where R = {(1,2), (2,2), (3,3),(1,2),(2,1), (2,3), (3,2)}

ii. Show that R is reflexive, symmetric but not transitive.

b) Determine whether the given relation is an equivalence relation on the give set.

i. $n \, R_1 m \leftrightarrow mn > 0$ and m,n in Z.

ii. $x \, R_2 \, y \leftrightarrow |x| = |y|$ and x, y in $\Re$.

iii. $x \, R_3 \, y \leftrightarrow |x - y| < 3$ and x,y in $\Re$.

Where $\Re$ is the set of all real numbers
Z is the set of all integers.

05. a) Suppose that f is a function whose domain is {a,b,c,d,e} and whose codomain is {1, 2, ….., n} where a,b,c,d and e are all distinct.

i. If f is onto, what can you say about n?
ii. If f is one to one, what can you say about n?
iii. If f has an inverse, what can you say about n?

b) Find $f^{-1}x)$ if it exists for the following functions.

i. $f(x) = 2x+3$; where $x \in \Re$
ii. $f(x) = |x| + 1$; where $-1 \leq x \leq 1$

c) Let f : A $\rightarrow$ B and g: B $\rightarrow$ C are one to one and onto functions.

i. Show that gof is one to one.

ii. Show that $(gof)^{-1}(x) = f^{-1}og^{-1}(x)$ (you may assume that gof is onto)

06. a) Determine whether the binary operations $*$ defined below are commutative or associative or both.

i. $*$ defined on Z by $a * b = a - b$

ii. $*$ defined on Z^+ by $a * b = 2^{ab}$

b) Prove that if $*$ is an associative and commutative binary operation on a set S, then $(a * b) * (c * d) = [(d * c) * a] * b$ for all $a,b,c,d, \in S$

c) Determine whether (G, $*$) is a group in the following cases.

i. $G = \Re$, $*$ denoted by $a * b = ab$ in $\Re$

ii. $G = \Re^+$, $*$ denoted by $a * b = \sqrt{ab}$ in $\Re^+$

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07. a) Iterate the recursive relationship $x_{n+1} = \lambda x_n(1 - x_n)$ for $\lambda = 2.8$ & indicate initial conditions $x_0 = 0.7$, for $0 < x < 1$ (at least 5 iteration steps are necessary).

b) Graphically show that for $\lambda = 2.8$, when n tends to $\propto$ x tends to some x^* value, taking $x_0 = 0.7$. Hence find x^*.

c) Draw the Bifurcation diagram & comment on the diagram when λ varies from 0 to 3.5.
Using the diagram verify that part (b) has one solution only.

08. A three dimensional system is governed by the following 3 differential equations.

$$\frac{dx}{dt} = 1 - 2y$$

$$\frac{dy}{dt} = -2x + 2z$$

$$\frac{dz}{dt} = 2y - z$$

And $t = 0$, $(x,y,z) = (0,1,1)$

Find the phase space value (x_n , y_n , z_n) for n = 1,2,3.

09. a) i. How many edges are there in a graph with 6 vertices each of which has degree 3.

ii. Is there a graph with degree sequence 1,2,2,3 on 4 vertices? Justify your answer.

b) i. Define tree and forest.

ii. If a forest F has 'c' components, 'v' vertices and 'e' edges, show that $v = e + c$.

c) Is there a simple bipartite graph with degree sequence 3,3,3,3,3,6,6,6,6,6,6,6,6,5?
Justify your answer.

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