THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
BACHELOR OF TECHNOLOGY (MECHANICAL) – LEVEL 5
FINAL EXAMINATION – 2008

MEX5233 – DYNAMICS OF MECHANICAL SYSTEMS

154

DATE : 14th MARCH 2009
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

A machine of mass m produces a disturbing force $P \sin \omega t$, where P is the magnitude and ω is the frequency. It is isolated from the ground by an isolator consisting of a spring of stiffness k and a damper having a damping ratio β.

(i) Show that the isolation factor (i) is given by

$$[i] = \left[\frac{1 + 4\beta^2 r^2}{(1 - r^2)^2 + 4\beta^2 r^2} \right]^{1/2}$$

where $r = \dfrac{\omega}{\omega_n}$ and $\omega_n = \sqrt{\dfrac{k}{m}}$.

You may assume that the linear displacement x of the machine at steady state is

$$x = \frac{P}{k} \frac{1}{\sqrt{(1 - r^2)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi), \text{ where } \phi = \tan^{-1}\left[\frac{2\beta r}{(1 - r^2)} \right].$$

(ii) Show that for effective isolation ($i < 1$), the frequency ratio, $r > \sqrt{2}$

(iii) A machine of mass 400 kg operates at a speed of 1500 rev/min. The machine is to be isolated from the ground by an isolator having a spring of stiffness 62.5 x 10^5 N/m. Determine whether this isolator is effective in isolating the force transmitted to the ground.

(iv) The isolator in (iii) above is replaced by an undamped isolator of spring stiffness k_1. Determine the value of k_1 which would limit the transmissibility ratio to 0.2.

QUESTION 02

Fig. Q2 shows two uniform rigid beams each of length $4a$ and mass $3m$ supported by four springs of stiffnesses k, k, k and $4k$. Both the beams are horizontal in the equilibrium position. Determine the equations of motion and obtain the frequency equation.

If m = 1 kg and k = 100 N/m, determine the frequencies and mode shapes.

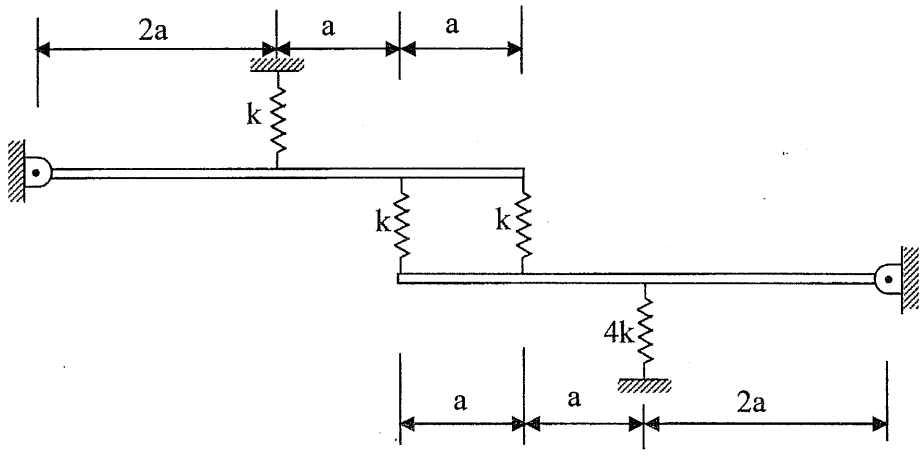


Fig. Q2

QUESTION 03

Fig. Q3 shows a torsional system consisting of rotors R_1 & R_2 and gears G_1 & G_2. The rotors and gears are connected by shafts S_1, S_2 and S_3 as shown. The number of teeth on gear G_2 is two times that on Gear G_1. The rotors, gears and shafts have the following parameters.

Moment of Inertia (kgm^2)		**Torsional Stiffness (Nm/rad)**	
Rotor R_1	6I	Shaft S_1	K
Rotor R_2	3I	Shaft S_2	3K
Gear G_1	I	Shaft S_3	2K
Gear G_2	2I		

Working from fundamentals, show that the frequency equation for natural frequencies of torsional oscillations of the system can be expressed as $(3\alpha - 2)(3\alpha^2 - 9\alpha + 1) = 0$,

where $\alpha = \dfrac{I\omega^2}{K}$ and ω is the natural frequency of torsional oscillations. Determine the natural frequencies in terms of K and I.

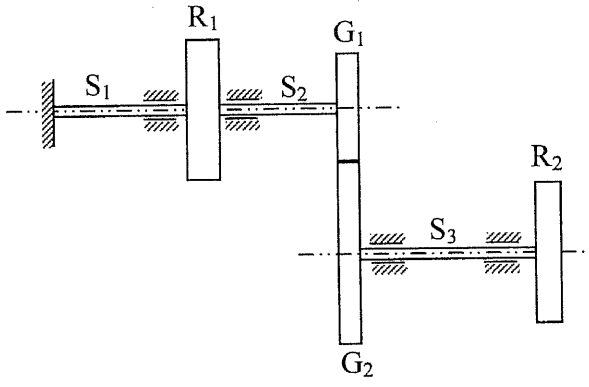


Fig. Q3

QUESTION 04

A uniform beam of mass *m* per unit length, length *L* and flexural rigidity *EI* is fixed at both ends. Starting from the equation,

$$\frac{\partial^4 y}{\partial x^4} + \frac{m}{EI}\frac{\partial^2 y}{\partial t^2} = 0$$

given in usual notation, for the transverse vibration of a beam with constant cross section, show that the frequency equation giving natural frequencies (ω) of transverse vibrations of the beam can be expressed as

$$\cos\lambda L.\cosh\lambda L - 1 = 0 \quad \text{where} \quad \lambda^4 = \frac{m\omega^2}{EI}.$$

QUESTION 05

Fig. Q5 shows a unity feedback control system. Obtain the closed loop transfer function and determine the natural frequency, damping ratio, damped natural frequency and peak time.

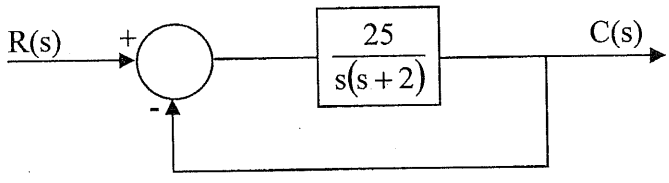


Fig. Q5

By introducing a PD controller having a transfer function $G_c(s) = 1 + T_d s$, to the above control system, the damping ratio is increased to 0.8. Determine the value of the derivative time T_d.

Determine the unit step response of the system with the PD controller. Assume zero initial conditions.

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K}{s(s^2+6s+13)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(ii) Using the root locus you have drawn show that $s = -4$ is a closed loop pole of the system when $K = 20$. Determine the other closed loop poles.

(iii) Determine, using the root locus you have drawn, the damping ratio of the closed loop system when $K = 20$.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{K}{s(1+0.1s)(1+s)},$$

where K is the gain of the controller.

Draw the Bode plots for $\frac{G(j\omega)}{K}$ and determine the value of K in order to have a

(i) gain margin of 10 db

(ii) phase margin of 30°.

QUESTION 08

Fig. Q8 shows an arrangement for controlling the temperature of a steam heated oven, utilizing temperature sensitive bellows and a hydraulic servomechanism. An increase in oven temperature θ causes the bellows to expand and the left end of the 'walking beam' ABC to move downwards, thus activating the hydraulic mechanism. The end C of the walking beam actuates a valve to reduce or increase the steam flow into the oven depending on the temperature in the oven. When x is zero the beam ABC whose mid point is B, is horizontal.

Given that,

(a) oil flow in the servomechanism per unit displacement $= Q$

(b) area of cross section of the piston in the working cylinder $= A$

(c) heat transfer across the walls of bellows to the thermo sensitive fluid in the bellows $= q = H(\theta - T)$

where T is the temperature of the fluid inside the bellows and H is a constant.

(d) rate of change of temperature in the bellows fluid $= Kq$

where K is a constant.

(e) extension rate of the bellows per unit temperature rise in the fluid in the bellows $= L$

Neglecting fluid compressibility, fluid leakage and inertia of moving parts draw a complete block diagram of the overall system and show that the transfer function relating the valve position y to oven temperature θ can be expressed as $\frac{L}{(1+\tau_1 s)(1+\tau_2 s)}$.

Find expressions for τ_1 and τ_2.

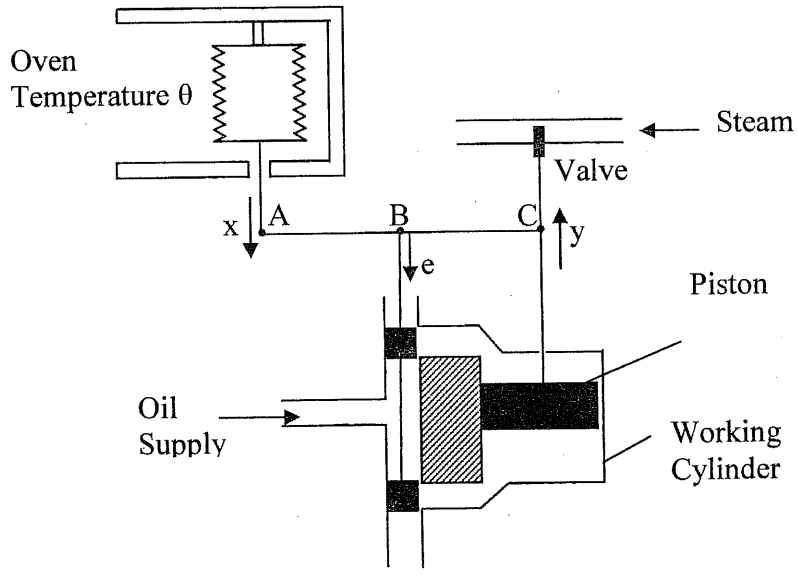


Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-.....-f^{n-1}(0)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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