THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 05

FINAL EXAMINATION – 2008/2009

MPZ 5230/MPZ 5330 – ENGINEERING MATHEMATICS III

DURATION : THREE (03) HOURS

301

Date : 16th March 2009 **Time: 0930 – 1230 hrs.**

Instructions

- **Answer only Six (06) questions.**
- **No. of pages in the paper - 05**
- **State any assumptions you use.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**
- **Request statistical table.**

01. a) Evaluate $\iint_R xy(x+y)dxdy$ over the region R bounded by $y = x^2$ and $y = x$.

b) Evaluate the surface integral of the function $e^{x^2+y^2+2}$ over the region bounded by $x^2 + y^2 \leq 4$, $x \geq 0$ and $y \geq 0$.

02. a) i. What you meant by solenoidal?

ii. Show that $3y^4z^2\underline{i} + 4x^3z^2\underline{j} - 3x^2y^2\underline{k}$ is solenoidal.

iii. Find the ρ such that the vector field $\underline{F} = (x+zy)\underline{i} + (y-2z)\underline{j} + (x+\rho z)\underline{k}$ is solenoidal.

b) i. Define the term irrotational.

ii. Show that $\underline{F} = (y+z)\underline{i} + (z+x)\underline{j} + (x+y)\underline{k}$ is irrotational.

iii. Find a scalar function ϕ such that $\underline{F} = \nabla\phi$, where $\underline{F}$ is defined in (ii).

03. (a) State Gauss divergence theorem.

(b) Verify divergence theorem for $\underline{F} = 4xy\underline{i} - y^2\underline{j} + yz\underline{k}$ taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

(c) Evaluate $\int_S \underline{F}.\underline{n}\, ds$, using the divergence; where $\underline{F} = yz\underline{i} + zx\underline{j} + xy\underline{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.

04. i. State the Stoke's Theorem.

ii. Verify Stoke's Theorem for $\underline{F} = (2x - y)\underline{i} - yz^2\underline{j} - y^2z\underline{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

05. a) Find the image of the region bounded by a circle of radius 2, a circle of radius 5, line $\theta = \frac{\pi}{3}$ and line $\theta = \frac{\pi}{2}$ under the mappings,

i. $w = z^2$ ii. $w = z^3$

b) Expand $f(z) = \frac{z+1}{(z-1)(z-3)}$ in a Laurent series for

i. $|z| < 1$

ii. $1 < |z| < 3$

iii. $|z-3| > 1$

06. a) Let $f(z) = z^3 - 10z^2 + 32z - 32$. Show that z = 2 is a root of f (z). Find the roots of f(z).

b) State the residue theorem.

c) Evaluate $\oint_C \frac{2z^2 - 15z + 30}{z^3 - 10z^2 + 32z - 32}\, dz$ for each of the following circles C,

i. $C: |z| = 3$ ii. $C: |z-4| = 1$

07. i. Find the Fourier integral representation of the function,

$$f(t) = \begin{cases} 0 & ; \ if \quad t < 0 \\ \frac{\pi}{2} & ; \ if \quad t = 0 \\ \pi e^{-t} & ; \ if \quad t > 0 \end{cases}$$

ii. Find the Fourier cosine transform of $f(x) = e^{-ax}$, $x > 0$ where a is a positive constant and hence evaluate $\int_0^\infty \frac{\cos ux}{a^2+u^2}\,du$

iii. Find the Fourier sine transform of $f(x) = xe^{-x}$

08. i) Let a transformation from a co-ordinate system x_i to a co-ordinate system x_i' be defined by $x_i' = \alpha_{ij} x_j$ where $\alpha_{ij} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} \end{pmatrix}$

(a) If $\underline{A} = -\boldsymbol{i}_1 + 2\boldsymbol{i}_2 - 2\boldsymbol{i}_3$, find its components in the x_i' co-ordinate system.

(b) If $\underline{B}' = 2\boldsymbol{i}_1' + 2\boldsymbol{i}_2' - 3\boldsymbol{i}_3'$, find its components in the x_i co-ordinate system.

ii) The moment of Inertia tensor of a certain body is given by

$$I_{ij} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

Show that in the co-ordinate system x_i^* defined by $x_i^* = \alpha_{ij} x_j$

$$(\alpha_{ij}) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{pmatrix}$$

$I_{11}^* = -2, \quad I_{22}^* = 1, \quad I_{33}^* = 4$

09. The components of a moment of inertia tensor are given by

$$(I_{ij}) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{pmatrix}$$

(a) Find the eigen values and the principal axes of the tensor.

(b) Express the tensor with respect to its principal axes.

(c) Write the equation of the transformation relating the co-ordinates x_i with that determined by the principal axes.

(d) Verify the results in (b) using the transformation law for second order tensor.

10. In a study of the effects on students for three different teaching methods, a group randomly assigned 30 of second grade students to teach in three of teaching methods (A, B, C). The results for the appropriate test as follows.

Do these data provide sufficient evidence to indicate that different teaching methods have different effects on students? $\alpha = 0.05$

Teaching Method	Test Scores									
A	24	16	16	16	10	15	19	17	25	13
B	13	09	08	08	09	15	15	14	14	13
C	10	03	00	03	09	10	07	10	02	09

11. A furniture company manufactures desks, tables and chairs. Each product needs the limited resources of lumber, carpentry and finishing; as described in the table. At most 5 tables can be sold per week. Maximize the weekly profit.

Resource	Desk	Table	Chair	Max. Availability
Lumber (board ft.)	8	6	1	48
Finishing hours	4	2	1.5	20
Carpentry hours	2	1.5	0.5	8
Max demand	Unlimited	1	Unlimited	5
Profit (in 100 Rupees)	60	30	20	

Then the linear programming model is formulated as;

Maximize $z = 60x_1 + 30x_2 + 20x_3$

Subject to $8x_1 + 6x_2 + x_3 \leq 48$

$$4x_1 + 2x_2 + 1.5x_3 \leq 20$$
$$2x_1 + 1.5x_2 + 0.5x_3 \leq 8$$
$$x_2 \leq 5$$

$x_1, x_2, x_3 \geq 0$

Where x_1, x_2 and x_3 be the number of desks, tables and chairs manufactured and the weekly profit be z (in 100 rupees). Obtain the optimal solution for above Linear programming model using **Simplex Method**.

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ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

ν_1 and ν_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 1%.

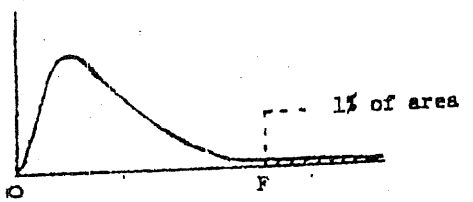


TABLE 01.4(C) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 1% TAIL)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

ν_1 and ν_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 5%.

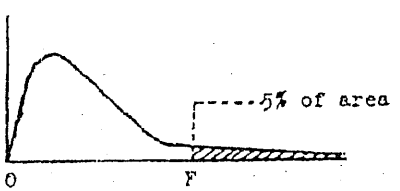


TABLE 0.4(A) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 5% TAIL)