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THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY – LEVEL 04
FINAL EXAMINATION – 2009/2010
MPZ 4230 – ENGINEERING MATHEMATICS II
DURATION: THREE (03) HOURS

Date : 16th March 2010 **Time: 0930-1230hrs.**

Instructions:

- Answer only six (06) questions.
- State any assumptions you required.
- Do not spend more than 30 minutes for any problem.
- Show all your workings.
- All symbols are in standard notation.
- Request statistical tables.

01. a) i. Given that $u(x,y)$ *and* $v(x,y)$ satisfy $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ *and* $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$. If $x=r\cos\theta$ and $y=r\sin\theta$, show that $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$

ii. If $u=x^4-6x^2y^2+y^4$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

b) If $F=(4xy-3x^2z^2)\underline{i}+2x^2\underline{j}-2x^3z\underline{k}$

i. Prove that F is conservative.
ii. Find ϕ such that $F=\nabla\phi$

c) A rectangle of the greatest area is inscribed in a triangle, whose base is a and height is h. Find the area of the above mentioned rectangle.

02. a) i. State the Cauchy-Riemann equations in polar form.

ii. Let $f(z)=z-a$ *and* $g(z)=\frac{1}{z-a}$, where z is a point on the circle $|z-a|=r$.
Check whether $(f(z))^n$ and $(g(z))^n$ satisfy the Cauchy-Riemann equations for all z in the circle $|z-a|=r$, where n is a positive integer.

b) Compute $\int_c (z-a)^n dz$ *and* $\int_c \frac{1}{(z-a)^n} dz$ for the following paths.

i. When c describes the circle $|z-a| = r$ in the positive sense.

ii. When c describes the square in the positive sense, with sides parallel to the axes, length 2a and having its center at the origin.

03. a) A large office has 100 personal computers. The probability that there will be a software problem for each computer during a particular day is 0.04.

i. What is the probability that more than 6 computers will have a software problem?
ii. Compute the mean and variance, assuming a binomial probability distribution.
iii. Compute the mean and variance, assuming a Poisson probability distribution. Compare the results with those of part (ii).

b) The miles per gallon for a specific model of a semi truck can be modeled by a normal distribution. The mean is 15 and the variance is 2.89. Determine the probability that the miles per gallon is

i. Less than 13
ii. Greater than 17.5
iii. Between 13.5 and 18
iv. A truck driver claimed that he attained 20 miles per gallon with a similar truck. What do you think about that claim?

04. a) A sample of 40 sales receipts from a grocery store has $\bar{x}$ = \$137 and σ = \$30.2. Use these values to test whether the mean of the sales at this grocery store are different from \$150.

b) The average amount of time some boys and girls age between 7 and 11 spend playing sports each day is believed to be the same. An experiment is done, data is collected as shown in the table below.

	Sample size	Average number of hours playing sports per day	Sample standard deviation
Girls	9	2 hours	$\sqrt{0.75}$
Boys	16	3.2 hours	1.00

Is there a difference in the average amount of time that the boys and girls aged between 7 and 11 play sports each day according to the above table? (Test at the 5% level of significance)

05. A random sample of 10 elementary school students are selected, and each student is measured on a creativity score (x), using a well defined testing instrument and on a task score (y), using a new instrument. The task score is the mean time taken to perform several hand-eye coordination tasks. The data are

Student	Creativity (x)	Tasks (y)
S1	28	4.5
S2	35	3.9
S3	37	3.9
S4	50	6.1
S5	69	4.3
S6	84	8.8
S7	40	2.1
S8	65	5.5
S9	29	5.7
S10	42	3.0

a) i. Plot the scatter diagram for those data on a graph paper.
ii. Using the above scatter diagram find the simple linear regression line.

b) Fit the simple linear regression model for the above data by using the theory of regression analysis (Without using the scatter diagram)

c) Compare the results of part (a) and part (b).

d) i. Is the simple linear regression model in part (b) is useful for the above mentioned random sample of 10 students.
ii. How well does the regression equation fit the data?

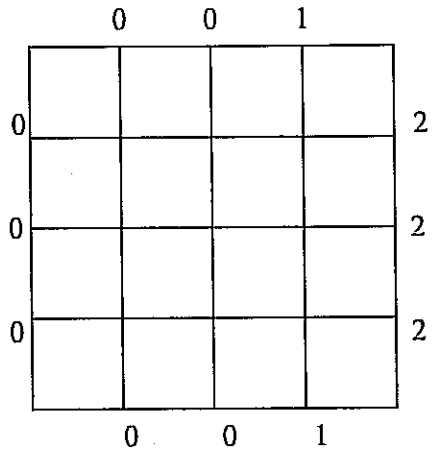
06. a) i. State and prove Euler's method for solving an ordinary differential equation.

ii. Find $y(0.1)$, $y(0.2)$ and $y(0.3)$, where $\frac{dy}{dx} = 1 - y$ *and* $y(0) = 1$

iii. Find the actual values of $y(0.1)$, $y(0.2)$ and $y(0.3)$.

b) Let $\frac{dy}{dx} = xy + y^2, y(0) = 1$. Find $y(0.1)$ and $y(0.2)$ by using the Taylor Series method.

07. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ where the boundary values are shown in the figure given below.



08. In the series RLC circuit, inductance (L) = 2 Henry, resistance (R) = 8 ohms and capacitance (C) = 10^{-1} farad and voltage (V_0) = 10 cos 2t volts.

a) Obtain an expression for the charge at any time t.

b) Using the above result find:

i. Capacitor charge at time T.

ii. Current at time T.

iii. Capacitor voltage at time T.

Kirchhoff's voltage law for series circuit yield.

$$V_L+V_R+V_C=V_{0,},$$

Where $V_L=L\frac{di}{dt},\ V_R=iR,\ V_C=\frac{1}{c}\int idt \quad and \quad i=\frac{dq}{dt}$

09. a) Write down the Fourier series representation of a function *f(x)* for $-L<x<L$ with period 2L .

b) Prove that the Fourier series representation of a 2-periodic function given on (-1, 1) by $f(t)=|t|$ is

$$f(t)=\frac{1}{2}-\frac{4}{\pi^2}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}\cos(2m+1)\pi t$$

c) Use part (b) to find the steady periodic solution to the differential equation $x''+10x=f(t)$ expressed as a finite series.

10. a) Define a subspace.

Determine which of the following sets are subspaces of $\mathbb{R}^3$

i. $\{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$

ii. $\{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2=1\}$

b) Determine whether each of the following sets of vectors are linearly independent or dependent.

i. (1, 0, 1), (-1, 0, -1)
ii. (1, 0, 1), (1, 1, 0), (2, 1, 1)
iii. (1, 1, 1), (1, 0, 0), (0, 1, 0)

c) Verify that the three vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$ form a basis for $\mathbb{R}^3$. Construct an orthogonal basis for $\mathbb{R}^3$ out of (u_1, u_2, u_3) by Gram-Schmidt process.

11. Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

a) Find the eigen values of A.

b) Find the unit eigen vectors of A

c) Obtain a matrix P such that P^TAP is a diagonal matrix.

d) If $S = P^TAP$, state the special properties of S^n and S^T.

e) Using the above results, reduce the quadratic form $Q(x)$ to form $Q(y)$, where

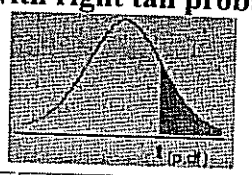
$Q(x) = 2x_1^2 + 2x_1x_2 + 2x_2x_3 + 3x_2^2 + 2x_3^2$

$Q(y) = a_1y_1^2 + a_2y_2^2 + a_3y_3^2$

f) Obtain the relationship between $x^T = (x_1, x_2, x_3)^T$ and $y^T = (y_1, y_2, y_3)^T$

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t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Probabilities for the standard normal distribution

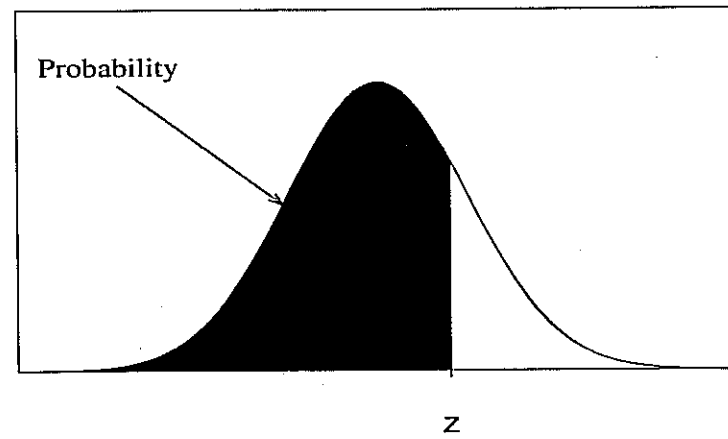


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998