THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
BACHELOR OF TECHNOLOGY (MECHANICAL) – LEVEL 5
FINAL EXAMINATION – 2009

MEX5233 – DYNAMICS OF MECHANICAL SYSTEMS

DATE : 06th MARCH 2010
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

A machine of mass m produces a disturbing force $P \sin \omega t$, where P is the magnitude and ω is the frequency. It is isolated from the ground by an isolator consisting of a spring of stiffness k and a damper having a damping ratio β. The motion of the machine is given, in usual notation, as

$$x = \frac{P}{m\omega_n^2} \frac{1}{\sqrt{\left(1-r^2\right)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi)$$

(i) Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T = \left[\frac{1+(2\beta r)^2}{\left(1-r^2\right)^2 + (2\beta r)^2} \right]^{1/2}$$

where $r = \dfrac{\omega}{\omega_n}$ and $\omega_n = \sqrt{\dfrac{k}{m}}$.

(ii) A machine of mass 800 kg and operating at a speed of 1200 rev/min. produces a repeating force of magnitude 16 kN. The machine is isolated from a rigid foundation by an isolator. Calculate the maximum stiffness of an undamped isolator which will limit the force transmitted to the foundation to 2400 N.

(iii) If the damping ratio of the isolator is 0.2, what would be the maximum stiffness of the isolator ?

(iv) What is the steady state amplitude of the machine if this isolator with the damping ratio of 0.2 is used ?

QUESTION 02

Fig. Q2 shows a uniform rigid rod of length $4a$ and mass $3m$ pivoted at its left end. The rod is supported by a spring of stiffnesses k attached to the rod at a point at a distance $3a$ from the left end. A spring of stiffness $2k$ and carrying a mass $2m$ is attached to the right end of the rod while another spring of stiffness k and carrying a mass m is attached to its mid point. The rod is horizontal in the equilibrium position. Determine the equations of motion for small oscillations of the system and show that the frequency can be expressed as

$$(\alpha - 1)(16\alpha^2 - 61\alpha + 9) = 0$$

where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of oscillations.

If $m = 2$ kg and $k = 2$ kN/m, evaluate the frequencies.

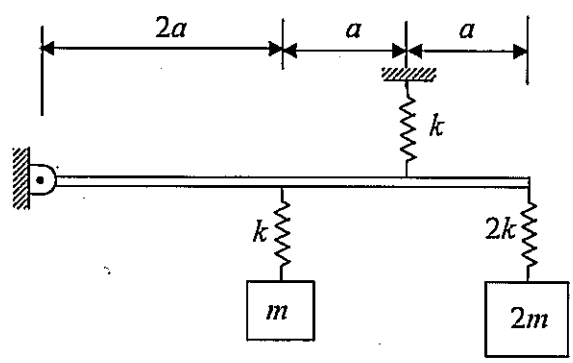


Fig. Q2

QUESTION 03

Fig. Q3 shows a system of five rotors R_1, R_2, R_3, R_4 & R_5 mounted on a shaft. Sections of the shaft are indicated as S_1, S_2, S_3 & S_4 and the parameters of the system are given in the Table below.

Polar Moments of Inertia of Rotors (kgm^2)	
R_1	8
R_2	10
R_3	16
R_4	10
R_5	15

Diameters & Lengths of Shaft Sections		
Section	Diameter (mm)	Length (mm)
S_1	20	100
S_2	30	100
S_3	25	110
S_4		120

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Using Holzer analysis determine the diameter of the section S_4 of the shaft if the system has a natural frequency of $(50/\pi)$ Hz. The modulus of rigidity of the material of the shaft is 80 GN/m^2.

Sketch the mode shape for the given frequency. Determine the position of the node between rotors R_1 & R_2.

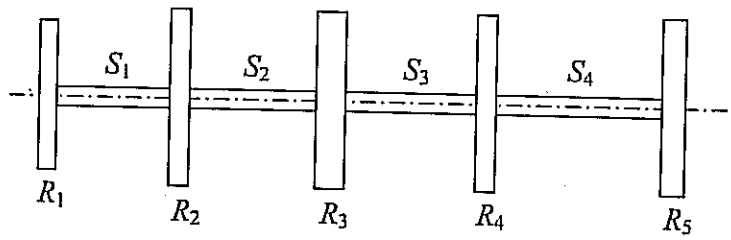


Fig. Q3

QUESTION 04

A uniform beam of mass m, length L and flexural rigidity EI is simply supported at each end. Determine the fundamental natural frequency of transverse vibration of the beam using Rayleigh's method. Assume that the vibrating form of the beam is $y = a\sin\left(\frac{\pi x}{L}\right)$ where y is the deflection of the beam at a distance x from one end and a is a constant.

Now a point mass m is placed on the beam at a distance b from one end. Determine the natural frequency of transverse vibration (Ω) of the beam, assuming the vibrating form to be the same.

Also show that Ω is a minimum when $b = \frac{L}{2}$

QUESTION 05

(i) Determine the damping ratio, undamped natural frequency and damped natural frequency for the speed control system in Fig. Q5(a). What is the response of this system to a unit step input when the initial conditions are zero ?

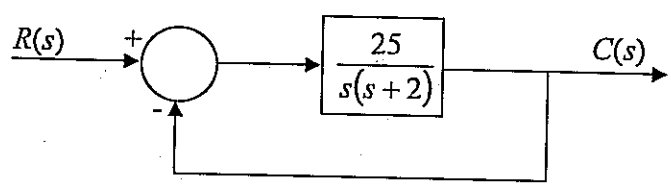


Fig. Q5(a)

(ii) To improve the dynamic performance of the speed control system shown in Fig. Q5(a), derivative action is added to the feed back path as shown in Fig. Q5(b). Determine the value of K such that the resulting system will have a damping ratio of 0.5. What is the unit step response of this system if all the initial conditions are zero?

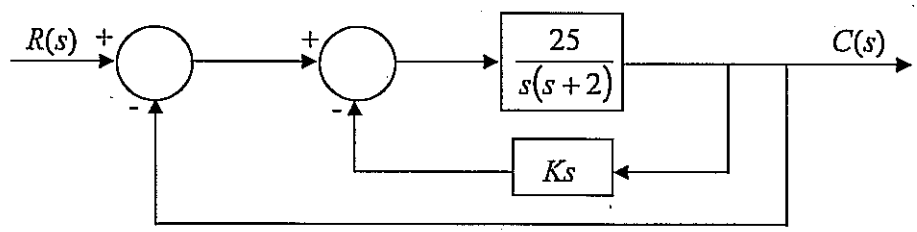


Fig. Q5(b)

(iii) Compare the peak time and percentage overshoot in (i) and (ii) above.

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K(s+6)}{s(s^2+6s+18)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and comment on the stability of the control system.

(ii) Using the root locus you have drawn show that $s = -2$ is a closed loop pole of the system when $K = 5$. Determine the other closed loop poles.

(iii) Determine, using the root locus you have drawn, the damping ratio of the closed loop system when $K = 5$.

(iv) Determine the unit impulse response of this system for $K = 5$. Assume that all initial conditions are zero.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{20}{s(s+2)(s+10)}.$$

Draw either a Bode plot or a Nyquist plot and determine the gain margin and the phase margin of the closed loop control system. Determine the factor by which gain has to be increased in order to have a gain margin of 6 db.

QUESTION 08

Fig. Q8 shows a hydraulic servomotor which is often used as a power amplifier in position control systems. The input x, the valve position e and the piston position y are connected together using a 'walking beam'. When $e = 0$, the valve is fully closed, the walking beam is horizontal and there is no oil flow to or from the slave cylinder. The cross sectional area of the piston is A and the oil flow to and from the slave cylinder (Q) is given by $Q = Ke$ where K is a constant. Assume oil to be incompressible.

(i) Draw a complete block diagram of the overall system and obtain the transfer function relating input (x) and output (y).

(ii) Show that when $a = b$, this transfer function reduces to $\frac{1}{1+Ts}$ where $T = \frac{2A}{K}$.

(iii) With the system initially at rest, x is given a unit step change. Obtain the variation of y with time and sketch a graph.

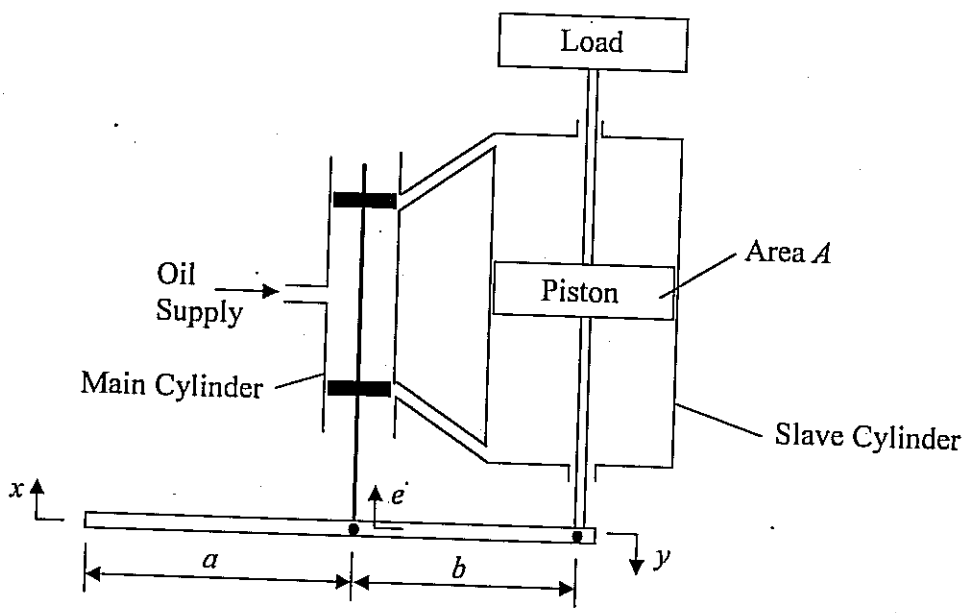


Fig. Q8

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LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-.....-f^{n-1}(0)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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