THE OPEN UNIVERSITY OF SRI LANAKA
BACHELOR OF TECHNOLOGY – LEVEL 05
FINAL EXAMINATION – 2009/2010
MPZ5230/MPZ5330 – ENGINERING MATHEMATICS III
DURATION: THREE (03) HOURS

Date: 09th March 2010 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer only six (06) questions.**
- **No of pages in the paper – 06.**
- **State any assumptions you use.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**

(01) (i) Use a double integral to find the area of the region R bounded by $xy = 1$ and $2x + y = 3$.

(ii) Write down the Cylindrical Polar Co-ordinates of the point $P = (x,y,z)$.
Compute the double integral $\iint_R \sin\left(\sqrt{x^2+y^2}\right)dxdy$ by transforming to the Cylindrical Polar Co-ordinates, where region enclosing the circle R is $x^2 + y^2 = \pi$.

(iii) Use spherical polar Co-ordinates to evaluate $\iiint_V z\sqrt{x^2+y^2+z^2}\,dv$, where V is the part of the unit ball $x^2 + y^2 + z^2 \le 1$ that lies in the first octant.

(02) (i) State the Gauss Divergence Theorem.

(ii) Prove that $\iint_S \underline{A}.\underline{n}ds = \iint_R \underline{A}.\underline{n}\frac{dydx}{|\underline{n}.\underline{k}|}$, where R is the projection of S on the x-y plane and $\underline{A}$ is a vector field.

(iii) Using the above part (ii) evaluate $\iint_S \underline{A}.\underline{n}ds$ where $\underline{A} = (x+y^2)\underline{i} - 2x\underline{j} + 2yz\underline{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

(iv) Use divergence theorem to evaluate $\iint_S \nabla r^2 ds$, where S is a closed surface enclosing the volume V and $\underline{r}$ is a position vector.

(03) (i)State the Stoke's Theorem.

(ii)If $\underline{a}$ is a constant vector and $\underline{r}$ is the position vector of any point except the ori then, show that $\text{Curl}(\underline{a}\times\underline{r}) = 2\underline{a}$.

(iii) Let S be a simple open surface bounded by the simple closed curve C. Show that $\oint_C (\underline{a}\times\underline{r})\,d\underline{r} = 2\iint_S (\underline{a}.\underline{n})\,ds$, here $\underline{n}$ is the unit outward normal to t surface S.

(iv) Let S_o be the hemispherical surface $x^2+y^2+z^2=a^2$, $z\geq 0$ and C be the boundar $x^2+y^2=a^2$, $z=0$. Using the above parts (ii) and (iii) evaluate $\oint_C (x\underline{j}-y\underline{i})\,d\underline{r}$.

(04) (i) Determine the equation of the curve in the w – Plane into which the straight line x = a is mapped under the following transformations, where a is positive re number.

(a) $w=z^2$ (b) $w=\frac{1}{z}$

(ii) Use (i) to sketch the images of the strip S in the z – plane bounded by the lines $x=\frac{1}{2}$ and $x=\frac{3}{4}$ under the transformations defined in above (a) and (b).

(iii) Find the radius of convergence of the series

$$1+\frac{(z-1)}{4}+\frac{(z-1)^2}{4^2}+\frac{(z-1)^3}{4^3}+\ldots\ldots\ldots$$

(iv) Find the Taylor series of the function $f(z)=\frac{1}{z+2i}$ with center at $z=i$.

(05) (i) Expand $\frac{1}{z-2}$ in Laurent's series for (a) $|z|<2$ (b) $|z|>2$.

(ii) Find Laurent's series expansion of $\frac{1}{(z+1)(z+3)}$ in powers of $(z+1)$ for the rang $0<|z+1|<2$.

(iii) (a) show that $z=4$ is a root of $p(z)=z^3-2z^2-7z-4$. Find the other roots of p(z).

(b) Hence determine the type of singularities of the function $\frac{z^2-2z}{z^3-2z^2-7z-4}$.

(c) Find the residues of the function $\frac{z^2-2z}{z^3-2z^2-7z-4}$.

(d) State residue theorem and evaluate $\oint_C \frac{z^2-2z}{z^3-2z^2-7z-4}dz$, where C is the circle with equation $|z|=3$.

(06) (i) Using dummy suffix notation, write out in full each of the following, where δ_{ij} is the kronecka delta.

(a) $\delta_{ij}a_j$ (b) $b_{ij}+c_{ij}$ (Note that i = 1,2,3 and j = 1,2,3)

(ii) Define the transformation law for vectors.

Let a transformation from a co-ordinate system x_i to a co-ordinate system x_i' be defined by the transformation law, where the direction cosines are in the matrix form $\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{2\sqrt{6}}{6} \end{pmatrix}$.

Let $\underline{A}=6\underline{i}_1+12\underline{i}_2+6\underline{i}_3$ in the x_i co-ordinate system and $\underline{B'}=\sqrt{3}\underline{i}'_1+\sqrt{2}\underline{i}'_2+\sqrt{6}\underline{i}'_3$ in the x_i' coordinate system.

(a) Determine the vector product of $\underline{A}$ and $\underline{B}$ in the x_i co-ordinate system.

(b) Determine the components of the vector product $\underline{A}$ and $\underline{B}$ in the x_i' co-ordinate system.

(iii) Let the components of a second order tensor be given by $(t_{ij})=\begin{pmatrix} 3 & 1 & -2 \\ 1 & 0 & 3 \\ 2 & -1 & 4 \end{pmatrix}$ and $\underline{a}_i$ of a vector be given by (2,-3,2). Find the inner product $t_{ij}a_j$.

(07) Consider the second –order symmetric tensor whose components are given by

$$(T_{ij}) = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix}$$

(a). Find the eigen values and the principal axes of the tensor.

(b). Express the tensor with respect to its principal axes.

(c). Write the equation of the transformation relating to the coordinates x_i with th determined by the principal axes.

(d). Verify the results in (b) by using the transformation law for second order tensor.

(08) (i) Define Fourier transformation and inverse Fourier transformation.

(ii) Show that $\bar{f}\{f(t-a)\} = e^{iwa}\bar{f}\{f(t)\}$, where $\bar{f}\{f(t)\}$ defines the Fourier transformation of f(t) and function of w.

(iii) Find the Fourier transformation of f(t), where $f(t) = \begin{cases} 1 & ; \ |t| < 1 \\ 0 & ; \ \text{Otherwise} \end{cases}$

Hence, obtain the Fourier transformation of $g(t) = \begin{cases} 1 & ; \ |t-3| < 1 \\ 0 & ; \ \text{Otherwise} \end{cases}$

(iv) Obtain the Fourier integral representation of the function

$$f(x) = \begin{cases} -1 & ; \ -1 < x < 0 \\ 1 & ; \ 0 < x < 1 \\ 0 & ; \ \text{Otherwise} \end{cases}$$

(09) A thin elastic string of length L has its ends fixed at the points x = 0 and x = L. If it is displaced from its equilibrium position, then its displacement u(x,t) satisfies the partial differential equation $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$ for $0 \le x \le L$.

(i) If u(x,t) = 0 at x = 0 and x = L for all values of t, use the method of separation of variables to determine a general solution for u.

(ii) If the initial displacement of the string is u(x,0) = f(x) ; where

$$f(x) = \begin{cases} \frac{2x}{L} & ; 0 \le x \le \frac{L}{2} \\ \frac{2(L-x)}{L} & ; \frac{L}{2} \le x \le L \end{cases}$$

and the string is released from rest , determine an expression for u(x,t).

(10) (a) Why analysis of variance is important?

(b) What are the assumptions in ANOVA?

(c) A Company wishes to purchase one of the three different machines A,B and C. An experiment is designed to test whether there are differences in the performance of their machines. Five experienced operators were assigned to each of the machine A,B and C for equal periods of time. The result of that experiment conducted by the operators is given below indicating the numbers of units produced by the machines.

A	68	72	77	42	53
B	72	53	63	53	48
C	60	82	64	75	72

Is there any evidence for differences in the performance of the machines at the level of significance 0.01?

(11) (i)A certain farm daily uses at least 800kg of special feed which is a mixture of c
and soybean meal. The following table provides the amount of protein and fiber in one
of corn and soybean meal.

Special feed	Protein	Fiber
corn	0.09	0.02
Soybean meal	0.60	0.06

The costs per kilogram of corn and soybean meal are Rs. 30 and Rs. 90 respectively. Tl
dietary requirements of the special feed are at least 30% protein and at most 5% fiber.

If the farm wishes to determine the daily minimum-cost feed mix, then

(a) Write down the decision variables that we seek to determine.

(b) Formulate the objective function and constraints of the problem.

(ii) (a)Reduce the following linear programming problem to the standard form.

Maximize $z = 5x_1 + 3x_2 + 4x_3$

subject to $2x_1 - 5x_2 \leq 6$

$2x_1 + 3x_2 + x_3 \geq 5$

$3x_1 + 4x_3 \leq 3$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(b) Solve the above LP problem by using the Two Phase method until Phase I is optimal. (Solving the problem up to the optimal solution in Phase I is enough and no need to solve the initial tableau of the phase II)

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ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

ν_1 and ν_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 5%.

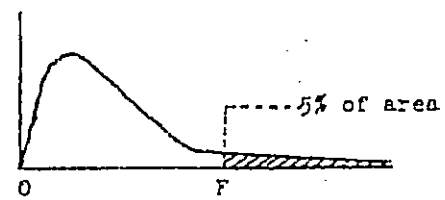


TABLE 01.4(A) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 5% TAIL)

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ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

ν_1 and ν_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 1%.

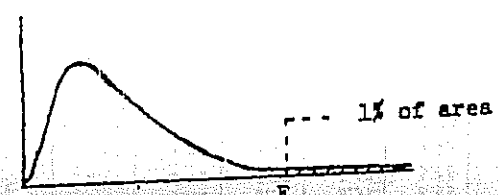


TABLE 01-4(C) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 1% TAIL)

TABLE 08.4(C) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 1% TAIL)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	405300	500000	540400	562500	576400	585900	592900	598100	602300	605600	610700	615800	620900	623500	626100	628700	631300	634000	636600
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.2	128.3	127.4	126.4	125.9	125.4	125.0	124.5	124.0	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.76	46.10	45.77	45.43	45.09	44.75	44.40	44.05
5	47.18	37.12	33.20	31.09	29.75	28.84	28.16	27.64	27.24	26.92	26.42	25.91	25.39	25.14	24.87	24.60	24.33	24.06	23.79
6	35.51	27.00	23.70	21.92	20.81	20.03	19.46	19.03	18.69	18.41	17.99	17.56	17.12	16.89	16.67	16.44	16.21	15.99	15.75
7	29.25	21.69	18.77	17.19	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.32	12.93	12.73	12.53	12.33	12.12	11.91	11.70
8	25.42	18.49	15.83	14.39	13.49	12.86	12.40	12.04	11.77	11.54	11.19	10.84	10.48	10.30	10.11	9.92	9.73	9.53	9.33
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.24	8.90	8.72	8.55	8.37	8.19	8.00	7.81
10	21.04	14.91	12.55	11.28	10.48	9.92	9.52	9.20	8.96	8.75	8.45	8.13	7.80	7.64	7.47	7.30	7.12	6.94	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.32	7.01	6.85	6.68	6.52	6.35	6.17	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.71	6.40	6.25	6.09	5.93	5.76	5.59	5.42
13	17.81	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.23	5.93	5.78	5.63	5.47	5.30	5.14	4.97
14	17.14	11.78	9.73	8.62	7.92	7.43	7.08	6.80	6.58	6.40	6.13	5.85	5.56	5.41	5.25	5.10	4.94	4.77	4.60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.54	5.25	5.10	4.95	4.80	4.64	4.47	4.31
16	16.12	10.97	9.00	7.94	7.27	6.81	6.46	6.19	5.98	5.81	5.55	5.27	4.99	4.85	4.70	4.54	4.39	4.23	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.05	4.78	4.63	4.48	4.33	4.18	4.02	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.87	4.59	4.45	4.30	4.15	4.00	3.84	3.67
19	15.08	10.16	8.28	7.26	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.70	4.43	4.29	4.14	3.99	3.84	3.68	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.56	4.29	4.15	4.00	3.86	3.70	3.54	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.44	4.17	4.03	3.88	3.74	3.58	3.42	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.33	4.06	3.92	3.78	3.63	3.48	3.32	3.15
23	14.19	9.47	7.67	6.69	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.23	3.96	3.82	3.68	3.53	3.38	3.23	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.14	3.87	3.74	3.59	3.45	3.29	3.14	2.97
25	13.88	9.22	7.45	6.49	5.88	5.46	5.15	4.91	4.71	4.56	4.31	4.06	3.79	3.66	3.52	3.37	3.22	3.06	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	3.99	3.72	3.59	3.44	3.30	3.15	2.99	2.82
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.92	3.66	3.52	3.38	3.23	3.08	2.92	2.75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.86	3.60	3.46	3.32	3.18	3.02	2.86	2.69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.80	3.54	3.41	3.27	3.12	2.97	2.81	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.75	3.49	3.36	3.22	3.07	2.92	2.76	2.59
40	12.61	8.25	6.60	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.40	3.15	3.01	2.87	2.73	2.57	2.41	2.23
60	11.97	7.76	6.17	5.31	4.76	4.37	4.09	3.87	3.69	3.54	3.31	3.08	2.83	2.69	2.55	2.41	2.25	2.08	1.89
120	11.38	7.32	5.79	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.78	2.53	2.40	2.26	2.11	1.95	1.76	1.54
∞	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.74	2.51	2.27	2.13	1.99	1.84	1.66	1.45	1.00

v_1 and v_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 0.1%.

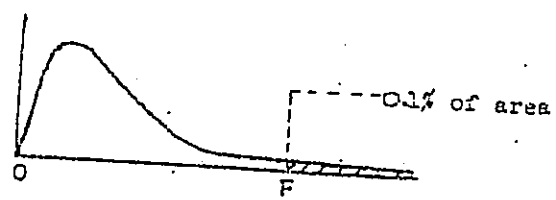


TABLE 08.4(B) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 1% TAIL)