00518

THE OPEN UNIVERSITY OF SRI LANAKA
BACHELOR OF TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2009/2010
MPZ 3231 – ENGINERING MATHEMATICS I A
DURATION: THREE (03) HOURS

Date: 14th March 2010 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer only six (06) questions selecting at least two questions each from sections A and B**
- **No. of pages in the paper – 06.**
- **Bold letters represent the vectors.**
- **Show all your workings.**
- **All symbols are in standard notation.**

SECTION – A

1. (a) If $\cos x\frac{dy}{dx}=y$, prove that $y^2=\lambda\left(\frac{1+\sin x}{1-\sin x}\right)$, where λ is an arbitrary constant.

   (b) Solve the differential equation $\cos x\frac{dy}{dx}+y\sin x=x^2\cos^2 x$.

   (c) If $\frac{dy}{dx}=(x+y-3)^2$, prove that $y=\tan(x+\mu)-x+3$, where μ is an arbitrary constant.

2. (a) Find the value of the constant k such that the differential equation $[3y^2+(k+1)x^2y]\frac{dy}{dx}-[(k-3)xy^2-3x^2]=0$ is exact. For the above value of k, solve the differential equation.

   (b) Prove that the solution of the differential equation $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+20y=0$ is $y=13e^{2x}\sin\left(4x+\tan^{-1}\frac{12}{5}\right)$, where $y(0)=12$ and $y^1(0)=44$.

P. T .O.

3. Applying row or column operations

(a) Solve the equation $\begin{vmatrix} a+x & b+x & c+x \\ b+x & c+x & a+x \\ c+x & a+x & b+x \end{vmatrix} = 0$.

(b) Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

(c) Find the value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$.

4. (a) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ show that $A^4 = I$, where I is the 3×3 unit matrix.

Find the period of A. Deduce A^{-1} and A^{2010}. Hence obtain the solution of the system

$$3x - 3y + 4z = 9$$
$$2x - 3y + 4z = 8$$
$$-y + z = 1.$$

(b) If $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, using the principle of mathematical induction prove that

$A^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$, where $n \in Z^+$.

Let $P = \begin{pmatrix} 5 & -2 \\ 7 & -4 \end{pmatrix}$, $R = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 1 \\ 7 & 1 \end{pmatrix}$

Prove that $Q^{-1}PQ = R$. Hence find P^n, where $n \in Z^+$.

P. T .O.

5. (a) Let $z,u,w \in C$ Prove that $|z+w|^2 = |z|^2 + |w|^2 + 2\text{Re}(z\bar{w})$. Hence prove that $|z+w|^2 = |z|^2 + |w|^2 + |u|^2 + 2\text{Re}(z\bar{w} + w\bar{u} + u\bar{z})$.

(b) State De Morvre's theorem

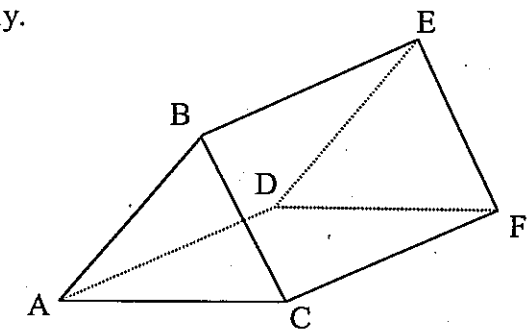
Find the values of $(-16)^{\frac{1}{4}}$. Write down the principle value of $(-16)^{\frac{1}{4}}$.

(c) Draw the paths of the following

(i) $\text{Im}(z^2) = 2$ (ii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ (iii) $\arg\left(\frac{1-3z}{1+i}\right) = \frac{\pi}{6}$.

6. Define the scalar (dot) product and the vector product of two vectors.

The triangular prism $ABCDEF$ has parallel triangular ends ABC and DEF, and the edges AD, BE and CF are parallel. The position vectors of the points A,B,C,D are $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \lambda\mathbf{k}$, $\mathbf{c} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{d} = 4\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ respectively.



(a) Show that A,B and C are non – collinear.

(b) Find the position vector of the point E in terms of λ.

(c) (i) Find $\vec{AB} \times \vec{AC}$.

(ii) Hence find $\vec{AD} \cdot \left(\vec{AB} \times \vec{AC}\right)$ in terms of λ.

(d) (i) Determine the volume of the prism $ABCDEF$ when $\lambda = 1$.

(ii) Describe the configuration of $ABCDEF$ when $\lambda = -4$.

P. T .O.

SECTION - B

7 (a) (i) Sketch the graph of $y = x^3 - 3x - 9$. Hence show that a real root of the equation $x^3 - 3x - 9 = 0$ lies between 2 and 3.

(ii) Using the bisection method

(I) Find the fifth iteration of the real root of the above equation.

(II) If the permissible error is $0 \cdot 5 \times 10^{-3}$, find the minimum number of iterations required to obtain the approximate root.

(b) Using Newton - Raphson method, establish the formula $x_{k+1} = \frac{1}{2}\left(x_k + \frac{N}{x_k}\right)$ to calculate the square root of N. Hence find the square root of 6 correct to four decimal places.

8. (a) Find the approximate value of $\int_0^{\pi} \sin x dx$

Using, (i) Trapezoidal rule (ii) Simpson's rule, by dividing the interval $[0, \pi]$ into six sub intervals.

Calculate the percentage errors from its true value in both cases.

(b) Using a suitable interpolation method and the following table, find $f(7.5)$

x	1	2	3	4	5	6	7	8
$f(x)$	2	9	28	64	120	202	280	315

P. T .O.

9. (a) Let A, B and C are three sets. Define $A \cup B$, $A \cap B$ and $A - B$

(i) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) Using algebra of sets prove that

(I) $(A \cap (A' \cup B)) \cup (B \cap (A' \cup B')) = B$,

(II) $A \cap ((B \cap C)' \cup (C \cap B'))' = A \cap B \cap C$.

(b) Define the axiomatic probability.

Let A, B and C are three events of a random experiment. Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Using the above formula, deduce the formula for $P(A \cup B \cup C)$.

(c) If $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{8}$ and $P(A') = \frac{5}{8}$ find

(i) $P(A)$ (ii) $P(B)$ (iii) $P(A' \cap B)$

(iv) $P(A \cap B')$ (v) $P(A' \cap B')$ (vi) $P(A' \cup B')$.

10. State and Prove Baye's theorem.

Three girls Nimali, Amali, Sumali pack biscuits in a factory. From a batch assigned to them Nimali packs 50% Amali 30% and Sumali 20%. The percentage that Nimali breaks some biscuits in a packet is 7% and respective percentages for Amali and Sumali are 9% and 11%. What is the probability that a packet with broken biscuits found by the checker was packed by Nimali? Who is the most probable person to be found with broken biscuits?

11. Two balls are drawn, without replacement from a box containing 3 red balls and 4 white balls.

The balls are drawn at random. X is the random variable for the number of red balls drawn.

(a) Find: (i) The probability density function of X.

(ii) The expected value of X.

(iii) The standard deviation of X.

(iv) The cumulative distribution function of X.

(b) Draw graphical representations of the probability density function and the cumulative distribution function.

P. T .O.

12. The continuous random variable X has the probability density function such that

$$f(x)=\begin{cases} \lambda(2+x)(4-x), & x\in[0,4] \\ 0, & x\notin[0,4] \end{cases}$$

(a) Find the value of the constant λ.

(b) Sketch the graph of $f(x)$.

(c) Calculate the expectation and the variance of the random variable X.

(d) Find the mode of the random variable X.

(e) Find the median of the random variable X.

(f) Find the cumulative distribution function of the random variable X.

END

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