00352

THE OPEN UNIVERSITY OF SRI LANAKA
BACHELOR OF TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2009/2010
MPZ 3230 – ENGINERING MATHEMATICS I
DURATION: THREE (03) HOURS

Date: 07th March 2010 **Time: 1330hrs – 1630hrs**

Instructions:

- **Answer only six (06) questions, selecting at least two questions from each section A and B**
- **No. of pages in the paper – 05.**
- **Bold letters represent the vectors.**
- **Show all your workings.**
- **All symbols are in standard notation.**

IMPORTANT INTEGRALS

- $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$
- $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$
- $\int p(x)e^{ax}dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - .....$
  sign alternate $(+-+-+-+-.....)$

SECTION - A

1 Find the general solutions of the following differential equations

(a) $\frac{dy}{dx} - 3\alpha y = x^2$, where α is a constant.

(b) $\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 32y = 12\sin 3x$.

2. (a) Using the definition of the Laplace transformation , find the Laplace transformation of $f(t)$, where

$$f(t) = \begin{cases} 1 & 0 < t < 4 \\ t^2 & 4 \le t \end{cases}$$

P. T .O.

(b) Define the convolution $f * g$ of the piecewise continuous functions $f(t)$ and $g(t)$.

Let $x(t) = e^t + 2\int_0^t \sin 2(t-u)x(u)du$. Find $x(t)$.

3. Using the Laplace transformation solve the following

(a) $\frac{d^2y}{dt^2} + y = 3\sin 2t$, where $t \geq 0$, $y(0) = 1$ and $y'(0) = 2$

(b) $\frac{dx}{dt} - 6x + 3y = 8e^t$

$\frac{dy}{dt} - 2x - y = 4e^t$, where $x(0) = 1$ and $y(0) = 0$.

4. (a) Using a trial function find the general solutions of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 6xe^{-x}$$

(b) If $\frac{dx}{dt} = 5x - 3y$ and $\frac{dy}{dt} = x + 2y$ find the characteristic equation of the above system of differential equations. Hence obtain the general solutions of the above system.

5. Using the matrices, discuss the solutions of the following systems

(a) $x + y = 2$

$x - y = 0$

$3x + y = t$, where $t \in R$.

(b) $x - 2y + 3z = 4$

$2x - 3y + az = 5$

$3x - 4y + 5z = b$, where $a, b \in R$.

P. T .O.

6. Applying row or column operations

(a) Express $\begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix}$ .as a product of factors.

(b) Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)(a+b+c)$.

SECTION – B

7. Define the scalar(dot) product and the vector product of two vectors

(a) The position vectors of the points A, B, C are given by $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$,

$\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$

(i) Find the value of λ such that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(ii) Find the area of the triangle ABC.

(b) Examine whether the vectors $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

are linearly independent or dependent.

(c) Find a unit vector perpendicular to both vectors $2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ and $3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$.

8. (a) If $\mathbf{A} = 2t\mathbf{i} - (t-1)\mathbf{j} + e^t\mathbf{k}$ and $\mathbf{B} = (t-1)\mathbf{i} + 2t\mathbf{j} - e^t\mathbf{k}$ Find $\frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$, $\frac{d}{dt}(\mathbf{A}\times\mathbf{B})$

and $\frac{d}{dt}(\mathbf{A}+\mathbf{B})$.

(b) If $\mathbf{r}$ is the position vector of a moving particle at time t prove that

$\frac{d}{dt}[\mathbf{r}\times(\mathbf{v}\times\mathbf{r})] = r^2\mathbf{a} + (\mathbf{r}\cdot\mathbf{v})\mathbf{v} - (r^2 + \mathbf{r}\cdot\mathbf{a})\mathbf{r}$, where $\mathbf{v}$ is the velocity and $\mathbf{a}$ is the

acceleration of the particle at time t

(c) If $\mathbf{f}(t) = \frac{\cos t}{\sqrt{1+e^{2t}}}\mathbf{i} + \frac{\sin t}{\sqrt{1+e^{2t}}}\mathbf{j} + \frac{e^t}{\sqrt{1+e^{2t}}}\mathbf{k}$ prove that

(i) $|\mathbf{f}(t)| = 1$. (ii) $\mathbf{f}(t)$ is perpendicular to $\frac{d}{dt}(\mathbf{f}(t))$.

P. T .O.

9. (a) Using a suitable interpolation method and the following table, find $f(15)$

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

(b) Show that a real root of the equation $xe^x - 2 = 0$ lies between 0 and 1. Using Newton Raphson method, find a real root of the equation $xe^x - 2 = 0$, correct to two decimal places.

(c)Using Lagrange's interpolation formula, find the polynomial passing through the points $(1,-7),(3,5),(4,8),(6,14)$.

10. (a) The events A and B such that $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{5}$ and $P(A|B') = \frac{11}{22}$.

Find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A'|B)$ (iv) $P(A|B)$

State whether A and B are

(I) independent, (II) mutually exclusive. Justify your answer.

(b) (i) A bag contains 5 red and 4 blue balls. Three balls are picked out, one at a time, and are not replaced. Find the probability that at least one of the three balls is blue.

(ii) A student attempts a multiple choice question to which 5 possible answers have been given and only one of them is correct. For any question there is a probability of $\frac{1}{3}$ that he knows the correct answer. If he does not know the correct answer, then he will mark one of the answers at random. He does, in fact, mark the correct answer. What is the probability that he knew the correct answer?

P. T .O.

11. Three dice are thrown. If the number 1 or 6 turns up, you will be paid1 *Rs*, but if neither the number 1 nor 6 turns up, you will pay5 *Rs*. How much would you expect to gain or lose in 9 games?

You are now given opportunity to change the rule for payment when the number 1 or 6 appears. To make the game worthwhile to yourself, what is the minimum amount in every day currency that you would suggest?

12. The time taken to fix a certain electronic item, t hours, has the probability density function

$$f(t)=\begin{cases} 10\lambda t^2 & 0\le t<0.6 \\ 9\lambda(1-t) & 0.6\le t\le 1.0 \\ 0 & otherwise \end{cases}$$,where λ is a constant.

(i) Find the value of λ

(ii) Sketch the graph of $f(t)$.

(iii)Write down the most likely time (mode).

(iv)Find the expected time.

(v) Determine the probability that the time will be

(I) more than 48 minutes, (II) between 24 and 48 minutes.

END

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