00109

THE OPEN UNIVERSITY OF SRI LANAKA
BACHELOR OF TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2009/2010
MPZ 3132 – ENGINERING MATHEMATICS I B
DURATION: THREE (03) HOURS

Date: 07th March 2010 **Time: 1330hrs – 1630hrs**

Instructions:

- **Answer only six (06) questions selecting at least two questions in each section A and B**
- **No. of pages in the paper – 06.**
- **Bold letters represent the vectors.**
- **Show all your workings.**
- **All symbols are in standard notation.**

IMPORTANT INTEGRALS

- $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$
- $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$
- $\int p(x)e^{ax}dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - .....$

  sign alternate $(+-+-+-+-.....)$

SECTION - A

1 (a) Define the Taylor polynomial of a function $f(x)$ about $x = a$. If $f(x) = \ln(1-x+x^2)$ prove that

$$(1-x+x^2)\frac{d^2f(x)}{dx^2} + (2x-1)\frac{df(x)}{dx} - 2 = 0.$$

Find the fifth order Taylor polynomial of $f(x)$ about $x = 0$.

(b) Using the Taylor series expansion, prove that $\ln(1-x) + \sum_{r=1}^{\infty}\frac{x^r}{r} = 0$, where the radius of convergence is $|x| < 1$.

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(i) Deduce that $\ln(1+3x-4x^2)=\sum_{r=1}^{\infty}\left((-1)^{r+1}4^r-1\right)\frac{x^r}{r}$ and the radius of convergence.

(ii) Deduce that $\ln 2=\sum_{r=1}^{\infty}\frac{1}{r2^r}$

2. (a) Using the definition of Laplace transformation, find the Laplace transformation of $f(t)$, where

$$f(t)=\begin{cases}1 & 0<t<4\\ t^2 & 4\le t\end{cases}$$

(b) Define the convolution $f*g$ of the piecewise continuous functions $f(t)$ and $g(t)$.

.Let $x(t)=e^t+2\int_0^t \sin 2(t-u)x(u)du$. Find $x(t)$.

3 Using the Laplace transformation solve the following:

(a) $\frac{d^2y}{dt^2}+y=2\sin 2t$ where $t\ge 0$, $y(0)=1$ and $y'(0)=2$

(b) $\frac{dx}{dt}-6x+3y=8e^t$ and $\frac{dy}{dt}-2x-y=4e^t$, where $x(0)=1$ and $y(0)=0$.

4 Prove that $\frac{1}{D-3\alpha}f(x)=e^{3\alpha x}\frac{1}{D}e^{-3\alpha x}f(x)$

Using the above result find the general solutions of the differential equation

$\frac{d^2y}{dx^2}-12\frac{dy}{dx}+32y=12\sin 3x$.

5 (a) Using a trial function find the general solutions of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=6xe^{-x}$$

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(b) If $\frac{dx}{dt} = 5x - 3y$ and $\frac{dy}{dt} = x + 2y$, then find the characteristic equation of the above system of differential equations. Hence obtain the general solutions of the above system.

6 (a) $f(x) = x^2$, $0 \le x \le 2\pi$ other values of x given by $f(x + 2k\pi) = f(x)$ where $k \in Z$. Sketch the graph of $f(x)$ Find the Fourier series representation of $f(x)$. Hence, deduce that $\sum_{r=1}^{\infty} \frac{1}{r^2} = \frac{\pi^2}{6}$.

(b) $g(x) = x(x - \pi)$, $0 \le x \le \pi$

Extend and sketch the function $g(x)$ as a

(i) Odd periodic function $x \in R$

(ii) even periodic function $x \in R$

(iii) Prove that $x(x - \pi) = \frac{\pi^2}{6} - \sum_{r=1}^{\infty} \frac{\cos 2rx}{r^2}$ and

$x(x - \pi) = \frac{8}{\pi} \sum_{r=1}^{\infty} \frac{\sin(2r-1)x}{(2r-1)^2}$ where $0 \le x \le 2\pi$.

SECTION - B

7 Using the matrices discuss the solutions of the following systems.

(a) $x + y = 2$

$x - y = 0$

$3x + y = t$, where $t \in R$

(b) $x - 2y + 3z = 4$

$2x - 3y + az = 5$

$3x - 4y + 5z = b$, where $a, b \in R$

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8 An engine of mass m moving on a horizontal straight line is working with constant power P and is subjected to resistance mkv^2 where k is a constant and v the speed. If the engine starts from rest and the maximum velocity is V, prove that $k = \frac{P}{mv^3}$. Prove that the engine acquires half of the maximum velocity after traveling a distance $\frac{1}{3k}\ln\left(\frac{8}{7}\right)$. Assuming the power is cut off and additional retarding force of constant value F is applied, find the subsequent time which elapses before the engine comes to rest.

9. (a) The position vector of a moving point P at any time t is

$\mathbf{r} = 5\sin 2t\,\mathbf{i} + 12\cos 2t\,\mathbf{j} + 3t\,\mathbf{k}$

(i). Describe the parth of P.

(ii) Find the velocity and acceleration vectors at any time t and also find the maximum values of their magnitudes.

(b) A force of magnitude $8N$ acting along the direction of $\vec{AB}$ from $C \equiv (2,3,-4)$ to $D \equiv (-4,2,2)$. where $A \equiv (4,3,6)$, $B \equiv (2,6,0)$ Find the work done by the above force.

(c) Two particles P and Q start simultaneously at the points $(-1,-1)$ and $(4,4)$ with constant velocities $2\mathbf{i}+\mathbf{j}$ and $\mathbf{i}-2\mathbf{j}$ respectively.

(i) Find the equations of their path in vector form.

(ii) Find the velocity of P relative to Q.

(iii) Obtain the expression for the distance between P and Q at any time t and hence find the minimum value of that.

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10. (a) A moving particle P on a plane has the polar coordinates of (r,θ) and the position vector is $\mathbf{r}$ at a time t. If $\boldsymbol{l}$ is the unit vector along the direction of $\mathbf{r}$ and $\boldsymbol{m}$ is the unit vector perpendicular $\mathbf{r}$ (with the direction of increasing θ)

Prove the following.

(i) $\boldsymbol{l} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$

(ii) $\boldsymbol{m} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$

(iii) $\dot{\boldsymbol{l}} = \dot{\theta}\boldsymbol{m}$ and $\dot{\boldsymbol{m}} = -\dot{\theta}\boldsymbol{l}$

(iv) The velocity $v = \dot{r}\boldsymbol{l} + r\dot{\theta}\boldsymbol{m}$ and the acceleration

$$\boldsymbol{f} = \left(\ddot{r} - r\dot{\theta}^2\right)\boldsymbol{l} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\boldsymbol{m}.$$

(b) A particle A of mass m moves on a smooth horizontal table and is connected by an inextensible string ,passing through a smooth hole O in the table to a particle B of mass M which moves along the vertical line through O. Initially B is at rest, $OA = a$ and A is projected horizontally with speed u perpendicular to OA. Show that the tension of the string is $\frac{mM}{M+m}\left(g + \frac{a^2u^2}{r^3}\right)$, where $OA = r$ at time t.

11 (i) Find the moments of inertia of a uniform circular ring of mass M with radius a, about the axis through the center, perpendicular to the plane of the ring.

(ii) Find the moments of inertia of a uniform circular disc of mass M with radius a about the axis through the center, perpendicular to the plane of the disc. Obtain the moments of inertia about a diameter of the above disc.

(iii) Find the moments of inertia of a uniform solid cone of height h and radius r about the line which is perpendicular to the axis passing through the vertex of the cone.

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(iv) The vertex of the cone is smoothly hinged at a point O. The cone is released from rest such that the axis of the cone is horizontal. If the cone moves such that the axis is in a vertical plane, find the reaction at O when the axis is vertical.

12 (a) If $\mathbf{A}=2t\,\mathbf{i}-(t-1)\mathbf{j}+e^{t}\mathbf{k}$ and $\mathbf{B}=(t-1)\mathbf{i}+2t\,\mathbf{j}-e^{t}\,\mathbf{k}$ find $\frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$, $\frac{d}{dt}(\mathbf{A}\times\mathbf{B})$ and $\frac{d}{dt}(\mathbf{A}+\mathbf{B})$.

(b) If $\mathbf{r}$ is the position vector of a moving particle at time t prove that $\frac{d}{dt}[\mathbf{r}\times(\mathbf{v}\times\mathbf{r})]=r^{2}\mathbf{a}+(\mathbf{r}\cdot\mathbf{v})\mathbf{v}-(r^{2}+\mathbf{r}\cdot\mathbf{a})\mathbf{r}$ where $\mathbf{v}$ is the velocity and $\mathbf{a}$ is the acceleration of the particle at time t.

(c) If $\mathbf{f}(t)=\frac{\cos t}{\sqrt{1+e^{2t}}}\mathbf{i}+\frac{\sin t}{\sqrt{1+e^{2t}}}\mathbf{j}+\frac{e^{t}}{\sqrt{1+e^{2t}}}\mathbf{k}$, prove that

(i) $|\mathbf{f}(t)|=1$ (ii) $\mathbf{f}(t)$ is perpendicular to $\frac{d}{dt}(\mathbf{f}(t))$.

END

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