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THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 06

FINAL EXAMINATION – 2010/2011

MPZ 6231 – DISCRETE MATHEMATICS

DURATION : THREE (03) HOURS

Date : 05th March 2011 **Time: 0930 - 1230 hrs.**

Instructions:

- **Answer only six questions**
- **State any assumption that you required**
- **Do not spend more than 30 minutes for one problem**
- **Show all your workings**
- **All symbols are in standard notation.**

01. i. State the inverse, the converse and the contra positive of the conditional statement $P \Rightarrow Q$.

ii. Construct the truth tables of the following compound statements.

a) $[(p \wedge q) \Rightarrow r] \Rightarrow [p \Rightarrow (q \Rightarrow r)]$

b) $[(p \vee q) \wedge (p \rightarrow r)] \rightarrow (p \vee r)$

iii. Prove using the contra positive: If the product of two integers x and y is odd, then both x and y are odd.

02. i. Prove that the statement, is true by using mathematical induction.

$$5+10+.....+5n=\frac{5n(n+1)}{2}$$

ii. Write down the duals of the following compound propositions.

a) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \Leftrightarrow \neg p$

b) $p \vee (p \wedge q) \Leftrightarrow p$

iii. Determine the truth values of the following compound proposition.

a) If 3 + 3 = 6 , then 3 x 3 = 9

b) If 3 + 3 = 33, then 3 x 3 = 9

c) If 3 + 3 = 33, then 3 x 3 = 33

d) If 3 + 3 = 6, then 3 x 3 = 33

03. i. Write these sets in a set builder form:

a) $A=\{2,4,5,8,.....\}$

b) $B=\{3\ \}$

ii. Let $U=\{a,b,c,d,e\}$, $A=\{a,b,d\}$ *and* $B=\{b,d,e\}$

Find

a) $B-A$

b) $A'\cap B$

c) $B'-A'$

iii. Without using venn diagram, show that

$(A\cup B)\setminus(A\cap B)=(A\setminus B)\cup(B\setminus A)$

iv. Explain the difference between $A\subseteq B$ *and* $A\subset B$

04. i. Let the function $f:R\to R$ be defined by

$$f(x)=\begin{cases}3x-1 & \text{if } x>3\\ x^2-2 & \text{if } -2\le x\le 3\\ 2x+3 & \text{if } x<-2\end{cases}$$

Find (a) $f(2)$ b) $f(5)$ c) $f(-1)$ d) $f(-4)$

ii. Let $W=\{-2,-1,-1,0,1,2\}$ and the function $f:W\to R$ be defined by the formula $f(x)=x^2+3$. Find the range of f.

iii. Let the functions $g:R\to R$ *and* $h:R\to R$ be defined by

$g(x)=3x-1$, $h(x)=x^2+3x+2$

Find formulas which define the product functions

a) *goh* b) *hog* c) *gog*

iv. Let $f:R^+\to R$ be defined by $f(x)=x^2+8$. Find a formula that defines f^{-1}.

05. i. Let $A=\{a,b\}, B=\{1,2\}$ *and* $c=\{2,3\}$

Find

a) $(A\times B)\cup(A\times C)$

b) $A\times(B\cap C)$

ii. Let $w=\{1,2,3,4\}$ *and* $R=\{(1,1),(1,3),(2,2),(3,1),(4,4)\}$. Is R reflexive?

iii. Let R be the relation in the natural numbers N defined by the open sentence "(x-y) is divisible by 5"; that is, let
$R=\{(x,y)/x\in N, y\in N,(x-y)$ is divisible by 5$\}$

Prove that R is an equivalence relation.

06. i. Define a group in usual notation and what is meant by an abelian group?

ii. Let G_1 be the set of all non zero real numbers and let
$a*b=\frac{ab}{7}$
Show that $(G_1,*)$ is an abelian group.

iii. Let $A=\{a,b,c\}$ and $*$ be the binary operation defined in the accompanying table. Determine the associative law and communities law of $*$

*	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

07. i. Draw the diagram of each of the following graphs $G=(V,E)$

a) $V=\{1,2,3,4,5,6\}$
$E=\{\{1,2\},\{1,3\},\{1,4\},\{2,5\},\{2,6\},\{3,5\},\{3,6\},\{4,5\},\{4,6\}\}$

b) $E=\{\{x,y\}, 2x+y$ *is even and* $x\le y\}$,
$V=\{1,2,3,4,5,6,7\}$

ii. Draw the graph with 5 vertices in which each vertices has degree 4.

iii. G is the graph whose adjacency matrix M is given by

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Without drawing a diagram of G determine whether G is connected or not.

iv. Draw the graph of adjacency matrix M is part (iii) above.

iv. Draw a tree with 8 vertices, exactly 4 of which have degree 1.

08. i. Iterate the $x_{n+1} = \lambda x_n(1-x_n)$ for n=0,1,2..... recursive relation for $\lambda = 3.2$, and indicate the initial condition x_0=0.9 (at least 8 interaction steps are necessary)

ii. Graphically show that the above system.

iii. Graphically show that $t_{n+1} + \lambda t_n^2 - \lambda t_n = 0$ for $\lambda = 2.7$ when n tends to ∞, x tends to some x^* value, taking $x_0 = -0.04$ and $x_0 = 0.04$.

09. A three dimensional system is governed by the following 3 differential equations.

$$\frac{dx}{dt} = 3x + 2y + 4z$$
$$\frac{dy}{dt} = 2x + 2z$$
$$\frac{dz}{dt} = 4x + 2y + 3z$$

At $t = 0$, $(x, y, z) = (0,1,1)$

Find the phase space values of (x_n, y_n, z_n) for $n = 1, 2$.

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