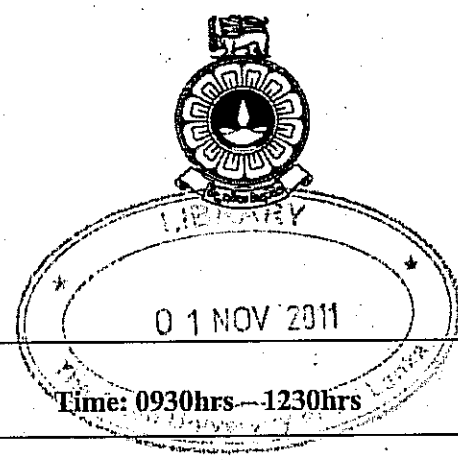
THE OPEN UNIVERSITY OF SRI LANAKA

BACHELOR OF TECHNOLOGY – LEVEL 05

FINAL EXAMINATION – 2010/2011

MPZ 5230 – ENGINERING MATHEMATICS III

DURATION: THREE (03) HOURS



Date: 08th March 2011 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer only six (06) questions.
- No of pages in the paper - 06
- State any assumptions you required.
- Do not spend more than 30 minutes for any problem.
- Show all your workings.
- All symbols are in standard notation.

(01) (i) Find the value of the surface integral of the function $f(x,y)=x^2+y^2$ over the region bounded by the curves $y=x$ and $y^2=4x$.

(ii) Write down the cylindrical polar co-ordinates of the point $P=(x,y,z)$

Use cylindrical polar co-ordinates to find the volume integral of the function $f(x,y,z)=x^2+y^2$ over the volume contained between two cylinders, $x^2+y^2=1$ and $x^2+y^2=4$, for $0\le z\le 2$

(iii) Write down the expression for grad f in terms of cylindrical polar co-ordinates.

Compute grad f in cylindrical- co-ordinates if f is given in Cartesian co-ordinates by $f(x,y,z)=\dfrac{z}{x^2+y^2}$

(iv) Use spherical polar co-ordinates to evaluate $\iiint xyz\,dxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$

(02) (i) State Gauss divergence theorem.

(ii) Verify the divergence theorem for $F = 4x\,\underline{i} - 2y^2\,\underline{j} + z^2\,\underline{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

(03) (i) State the Stoke's theorem

(ii) Verify Stoke's theorem for $\underline{F} = y^2\,\underline{i} + xy\,\underline{j} + xz\,\underline{k}$, over the surface of hemisphere $x^2 + y^2 + z^2 = 1$ above the xy plane.

(04) (i) Find the image of the right half plane $Re(z) = x > 1$ under the linear transformation $w = (-1+i)\,z - 2 + 3\,i$ and sketch it.

(ii) Discuss the way in which the z – plane is mapped on the w- plane by the function $w = z^2$.

(a) Hence deduce the image of the lines $x = a$ and $y = c$.

(b) By using part (a) find the image region of the rectangular region bounded by the lines $x = \frac{1}{2}, x = 1, y = \frac{1}{2}, y = 1$ and sketch it.

(iii) Expand the following function in Laurent's series $\dfrac{z}{(z+1)(z+2)}$ about $z = -2$

(05) (i) (a) Find the roots of the $g(z) = z^3 + z^2 - 8z - 12$

(b) Find the partial fractions of the function $f(z) = \dfrac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$

(c) Find the Taylor series of the function $f(z)$ with center $z_0 = 1$

(d) Test the convergence of the above Taylor series

(ii) (a) Consider the function $f(z)=\frac{e^z}{(z-5)(z^2-4)}$ and identify the singularities of $f(z)$

(b) Find all the residues of the function $f(z)$

(c) Evaluate the integral $\int_c f(z)\,dz$ where c is the contour $|z|=3$.

(06) (i) Define the following terms.

(a) Sum of two tensors (b) Contraction of a tensor.

(ii) Let $A=(1,4,6)$, $B=(5,2,3)$, $C=\begin{pmatrix}0 & -1 & 3\\ 1 & 0 & 2\\ -3 & -2 & 0\end{pmatrix}$ and $D=\begin{pmatrix}1 & 3 & 5\\ 4 & 1 & 2\\ -3 & -2 & 1\end{pmatrix}$

be four tensors.

(a) Out of the above 4 (A, B, C and D) tensors, state the couples, that can be summed and write the order of those

(b) Find Contraction of the tensor C and D

(iii) Define the transformation law for vectors

Let a transformation from a co-ordinate system x_i to a co-ordinate system x_i' be defined by the transformation law, where the direction cosines are in the matrix form

$$\alpha_{ij}=\begin{pmatrix}1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3}\\ 1/\sqrt{2} & 0 & -1/\sqrt{2}\\ 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6}\end{pmatrix}$$

Let $\underline{A} = 3\,\underline{i}_1 + 2\,\underline{i}_2 + 6\,\underline{i}_3$ in the x_i co-ordinate system and $\underline{B}' = \sqrt{3}\,\underline{i}_1' + \sqrt{2}\,\underline{i}_2' + \sqrt{6}\,\underline{i}_3'$ in the x_i' co-ordinate system.

Determine the components of the vector product $\underline{A}$ and $\underline{B}$ in the x_i' co - ordinate system.

(07) Consider the second order symmetric tensor whose components are given by;

$$T_{ij} = \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

(i) Find the eigen values and the principal axes of the tensor.

(ii) Express the tensor with respect to its principal axes

(iii) Write the equation of the transformation relating to the co-ordinates x_i with that determined by the principal axes.

(iv) Verify the results in (ii) by using the transformation law for second order tensor.

(08) The vibrations of an elastic string of length L are governed by the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

The string is fixed at the ends. $u(0,t) = 0 = u(L,t)$ for all t.

The initial deflection is $u(x,0) = 2\,\mathrm{Sin}\frac{\pi x}{L} + 5\,\mathrm{Sin}\frac{3\pi x}{L}$ and the initial velocity is zero. Find the deflection of a string at any instant of time.

(09) (i) Define Fourier transformation and inverse Fourier transformation.

(ii) Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$$

(iii) (a) Write the Fourier sine transform and Fourier cosine transform of a function.

(b) Find the Fourier cosine transforms of e^{-ax}. Hence show that

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + 1} d\lambda = \frac{\pi}{2} e^{-x}; x \geq 0$$

(iv) The Fourier Integral theorem states that

$$f(x) = \int_0^\infty \{A(\omega)\text{Cos}\omega x + B(\omega)\text{Sin}\omega x\} d\omega$$

Where $A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)\ \text{Cos}\omega x\ dx$ and $B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)\ \text{Sin}\omega x\ dx$

Then show that $f(x) = \frac{1}{\pi} \int_{\omega=0}^{\infty} \int_{u=-\infty}^{\infty} f(u)\ \text{Cos}\ \omega(x-u)\ du d\omega$ is equivalent form of Fourier's integral theorem.

(10).The following data represents the number of units of producing per day turned out by 5 different workmen using different types of machines

Workmen	Machines Type			
	A	B	C	D
1	44	38	47	36
2	46	40	52	43
3	34	36	44	32
4	33	38	46	33
5	38	42	49	39

(i) Explain briefly the technique of analysis of variance of data with one way classification.

(ii) Test whether the mean productivity is the same for the different machine type.

(11) Consider the following Linear Programming Problem:

Minimize $z = 60x_1 + 40x_2$

subject to the constraints

$30x_1 + 10x_2 \geq 240$

$10x_1 + 10x_2 \geq 1600$

$20x_1 + 60x_2 \geq 480$

where $x_1, x_2 \geq 0$

(i).What is the optimal solution to this problem? Solve it graphically.

(ii).Write down dual problem as maximization.

(iii) Solve the part (ii) by using simplex method.

(iv) Compare optimal solution of part (i) and part (ii).

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ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

ν_1 and ν_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 5%.

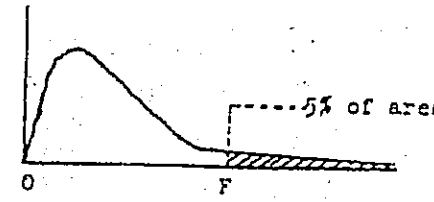


TABLE 01.4(A) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER 5% TAIL)

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v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	405300	500000	540400	562500	576400	585900	592900	598100	602300	605600	610700	615800	620900	623500	626100	628700	631300	634000	636600
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.2	128.3	127.4	126.4	125.9	125.4	125.0	124.5	124.0	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.76	46.10	45.77	45.43	45.09	44.75	44.40	44.05
5	47.18	37.12	33.20	31.09	29.75	28.84	28.16	27.64	27.24	26.92	26.42	25.91	25.39	25.14	24.87	24.60	24.33	24.06	23.79
6	35.51	27.00	23.70	21.92	20.81	20.03	19.46	19.03	18.69	18.41	17.99	17.56	17.12	16.89	16.67	16.44	16.21	15.99	15.75
7	29.25	21.69	18.77	17.19	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.32	12.93	12.73	12.53	12.33	12.12	11.91	11.70
8	25.42	18.49	15.83	14.39	13.49	12.86	12.40	12.04	11.77	11.54	11.19	10.84	10.48	10.30	10.11	9.92	9.73	9.53	9.33
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.24	8.90	8.72	8.55	8.37	8.19	8.00	7.81
10	21.04	14.91	12.55	11.28	10.48	9.92	9.52	9.20	8.96	8.75	8.45	8.13	7.80	7.64	7.47	7.30	7.12	6.94	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.32	7.01	6.85	6.68	6.52	6.35	6.17	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.71	6.40	6.25	6.09	5.93	5.76	5.59	5.42
13	17.81	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.23	5.93	5.78	5.63	5.47	5.30	5.14	4.97
14	17.14	11.78	9.73	8.62	7.92	7.43	7.08	6.80	6.58	6.40	6 13	5.85	5.56	5.41	5.25	5.10	4.94	4.77	4 60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.54	5.25	5.10	4.95	4.80	4.64	4.47	4.31
16	16.12	10.97	9.00	7.94	7.27	6.81	6.46	6.19	5.98	5.81	5.55	5.27	4.99	4.85	4.70	4.54	4.39	4.23	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.05	4.78	4.63	4.48	4.33	4.18	4.02	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.87	4.59	4.45	4.30	4.15	4.00	3.84	3 67
19	15.08	10.16	8.28	7.26	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.70	4.43	4.29	4.14	3.99	3.84	3.68	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.56	4.29	4.15	4.00	3.86	3.70	3.54	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.44	4.17	4.03	3.88	3.74	3.58	3.42	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.33	4.06	3.92	3.78	3.63	3.48	3.32	3 15
23	14.19	9.47	7.67	6.69	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.23	3.96	3.82	3.68	3.53	3.38	3.23	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.14	3.87	3.74	3.59	3.45	3.29	3.14	2 97
25	13.88	9.22	7.45	6.49	5.88	5.46	5.15	4.91	4.71	4.56	4.31	4.06	3.79	3.66	3.52	3.37	3.22	3.06	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	3.99	3.72	3.59	3.44	3.30	3.15	2.99	2 82
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.92	3.66	3.52	3.38	3.23	3.08	2.92	2 75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.86	3.60	3.46	3.32	3.18	3.02	2.86	2 69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.80	3.54	3.41	3.27	3.12	2.97	2.81	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.75	3.49	3.36	3.22	3.07	2.92	2.76	2.59
40	12.61	8.25	6.60	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.40	3.15	3.01	2.87	2.73	2.57	2.41	2.23
60	11.97	7.76	6.17	5.31	4.76	4.37	4.09	3.87	3.69	3.54	3.31	3.08	2.83	2.69	2.55	2.41	2.25	2.08	1.89
120	11.38	7.32	5.79	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.78	2.53	2.40	2.26	2.11	1.95	1.76	1.54
∞	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.74	2.51	2.27	2.13	1.99	1.84	1.66	1.45	1.00

v_1 and v_2 are the number of degrees of freedom in the numerator and denominator respectively.

The probability of F exceeding the entry value is in all cases 0.1%.

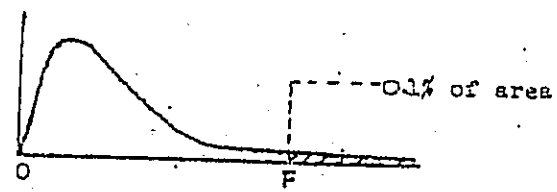


TABLE 08.4(B) : PERCENTAGE POINTS OF THE F DISTRIBUTION (UPPER .1% TAIL)

(iii)

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