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THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
BACHELOR OF TECHNOLOGY (MECHANICAL) – LEVEL 5
FINAL EXAMINATION – 2010

MEX5233 – DYNAMICS OF MECHANICAL SYSTEMS

DATE : **06th MARCH 2011**
TIME : **0930 HRS. – 1230 HRS.**
DURATION : **THREE (03) HOURS**

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

A machine of mass m is mounted on the ground through an isolator consisting of a spring of stiffness k and a damper having a damping ratio β. The machine produces an excitation force of the form $P \sin \omega t$, where P is the magnitude and ω is the frequency. The displacement of vibratory motion of the machine (x) at steady state is given as

$$x = \frac{P}{m\omega_n^2} \frac{1}{\sqrt{(1-r^2)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi) \text{ where } r = \frac{\omega}{\omega_n}, \quad \omega_n = \sqrt{\frac{k}{m}} \text{ and } \phi = \tan^{-1}\left(\frac{2\beta r}{1-r^2}\right).$$

Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T = \left[\frac{1+(2\beta r)^2}{(1-r^2)^2 + (2\beta r)^2}\right]^{1/2}$$

A compressor of mass m is mounted on a concrete foundation of mass M in the shop floor of a factory as shown in Fig. Q1(a). The soil provides isolation with negligible damping to the system. A simplified model of the system is shown in Fig. Q1(b) in which k is the equivalent stiffness of the soil and $P \sin \omega t$ is the unbalanced force generated by the compressor running at an angular speed ω. If the isolation reduces the transmitted force to the ground by 75%, determine the height of the concrete foundation required using information given below.

$M = 800$ kg

$k = 2.0 \times 10^7$ N/m

Steady state operating speed of the compressor – 1,800 rev/min.

Density of concrete – 2,300 kg/m^3

Surface area of the concrete block – 4 m^2

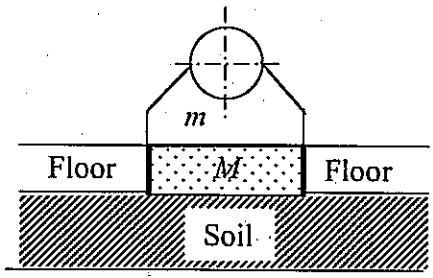


Fig. Q1(a)

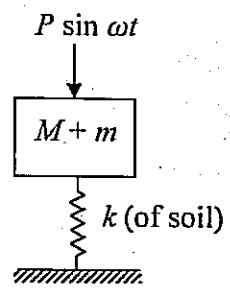


Fig. Q1(b)

QUESTION 02

A uniform rigid rod of length $3a$ and mass m is pivoted at a point at a distance a from the left end. The rod is supported by two springs each of stiffnesses k, attached to the rod at the left and right ends. A spring of stiffness k is attached to the rod at a point at a distance $2a$ from the left end and this spring carries a mass m to which another spring of stiffness $6k$ and another mass m are attached. This arrangement is illustrated in Fig. Q2. The rod is horizontal in the equilibrium position. Determine the equations of motion for small oscillations of the system and show that the frequency equation can be expressed as

$$(\alpha - 6)(\alpha^2 - 13\alpha + 5) = 0$$

where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of oscillations.

If $m = 1$ kg and $k = 1$ kN/m, evaluate the frequencies.

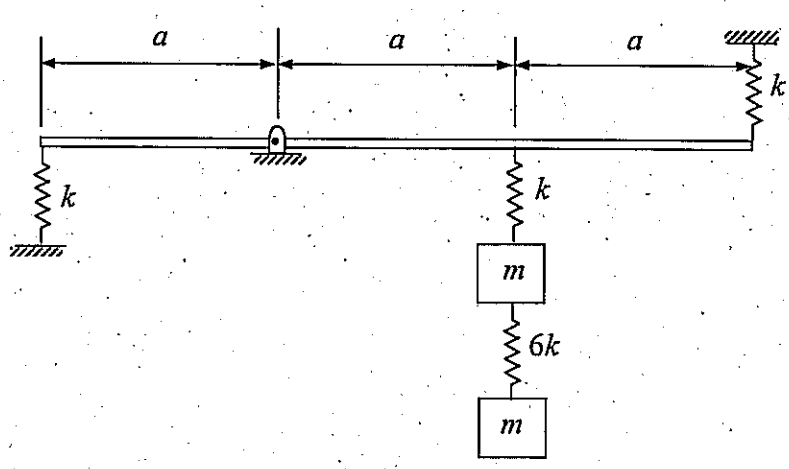


Fig. Q2

QUESTION 03

A light elastic shaft AB of uniform diameter, is supported freely in bearings. The shaft carries a wheel at each end and it is found that the natural frequency of torsional vibration is 40 Hz. A third wheel is mounted on the shaft at a point C such that AC = ¾ AB. If all the wheels have the same moment of inertia, determine the natural frequencies of torsional vibration.

QUESTION 04

A uniform beam of flexural rigidity EI and a total mass m over a length L is simply supported at each end. The dynamic deflection curve is similar to the static deflection curve. Determine the fundamental natural frequency of transverse vibration of the beam using Rayleigh's method.

Determine the natural frequency of transverse vibration of a steel beam of diameter 30 mm, length 2 m, simply supported at the ends and carrying a uniformly distributed load of 50 kg/m. Assume that the mass of the beam too is uniformly distributed over the span of the beam.

Modulus of elasticity of steel – 200 GN/m^2
Density of steel – 7.8 x 10^3 kg/m^3

QUESTION 05

(i) The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(sT+1)}$$

By what factor the gain K should be changed in order to increase the damping ratio from 0.4 to 0.8.

(ii) Determine the damping ratio, undamped natural frequency, damped natural frequency and the settling time for the control system in Fig. Q5(a).

A controller is added to this control system, as shown in Fig. Q5(b) in order to increase the damping ratio to 0.8. Determine the value of derivative time T_d.

Obtain the unit impulse response of the system for this value of T_d. Assume zero initial conditions.

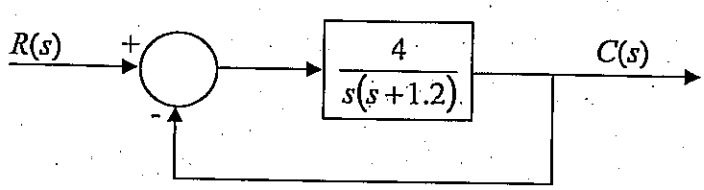


Fig. Q5(a)

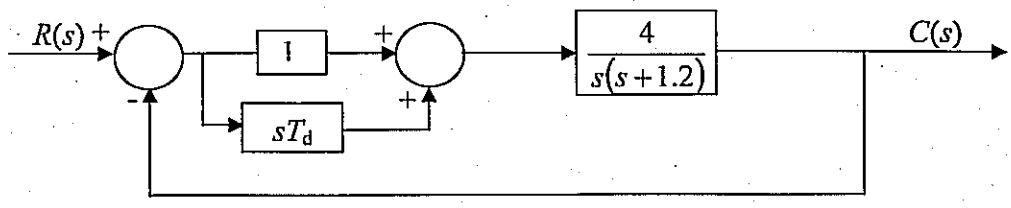


Fig. Q5(b)

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K}{(s+2)(s+4)(s+6)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(ii) Using the root locus you have drawn show that $s = -8$ is a closed loop pole of the system for $K = 48$. Determine the other closed loop poles.

(iii) Determine the damping ratio of the closed loop system when $K = 48$.

(iv) Determine the unit impulse response of this system for $K = 48$. Assume that all initial conditions are zero.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{K}{s(s+5)(s+50)}.$$

Draw either a Bode plot or a Nyquist plot for $K = 2500$ and determine the gain margin and the phase margin of the closed loop control system.

Also determine the open loop gain for

(i) a gain margin of 8 db

(ii) a phase margin of 15°.

QUESTION 08

Fig. Q8 shows a schematic diagram of a pneumatic proportional controller. When the actuating error (e) is zero an equilibrium state exists with the flapper amplifier distance equals to $\overline{X}$, the displacements of the bellows and diaphragm equal to $\overline{Y}$ and $\overline{Z}$ respectively, the nozzle back pressure equals to $\overline{P}_b$ and the control pressure equals to $\overline{P}_c$. x, y, z, p_b and p_c are the deviations of the above parameters from the equilibrium position when an error e exists. The equivalent spring constant and the effective area of the bellows are K_1 and A respectively and the stiffness of the spring attached to the bellows is K_2.

Assuming linear relationships between, p_b & x, p_b & z and p_c & z, draw a complete block diagram of the overall system and obtain the transfer function $\frac{P_c(s)}{E(s)}$. You may assume suitable proportionality constants.

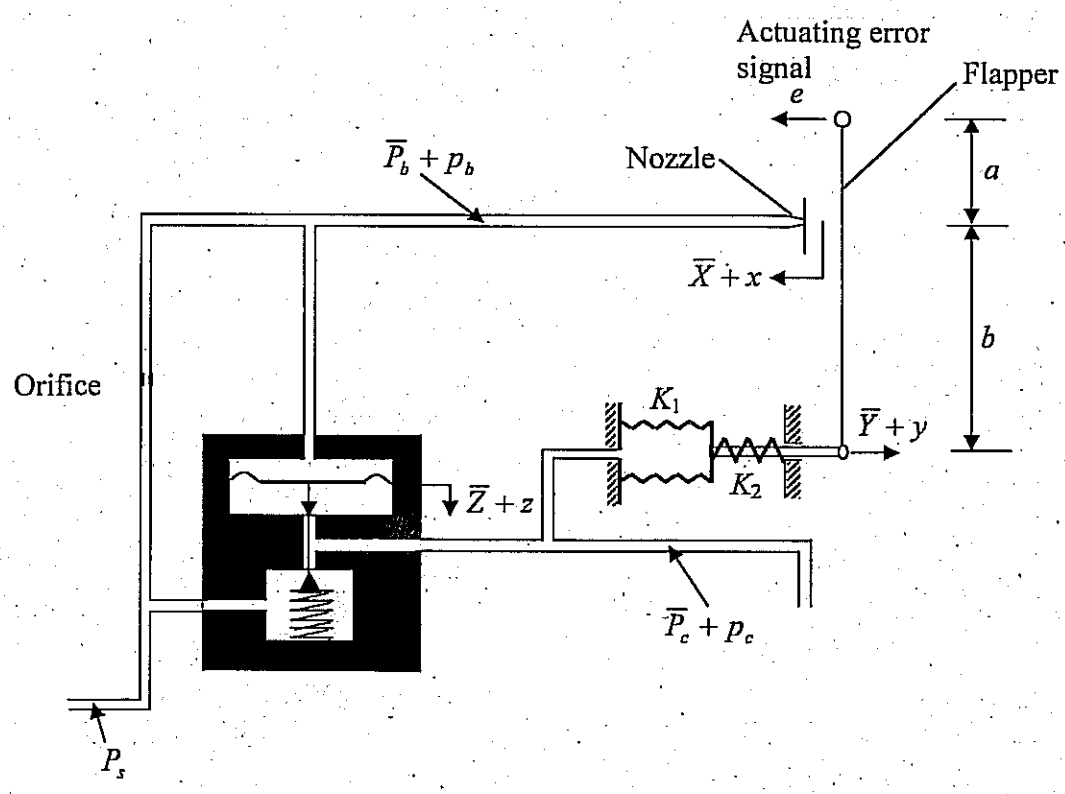


Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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