THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
BACHELOR OF TECHNOLOGY (MECHANICAL) – LEVEL 5
FINAL EXAMINATION – 2011

MEX5233 – DYNAMICS OF MECHANICAL SYSTEMS

DATE : **26th FEBRUARY 2012**
TIME : **0930 HRS. – 1230 HRS.**
DURATION : **THREE (03) HOURS**

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

A machine of mass m produces a disturbing force $P \sin \omega t$, where P is the magnitude and ω is the frequency. The machine is isolated from the ground by an isolator consisting of a spring of stiffness k and a damper having a damping ratio β. The linear displacement of the machine (x) at steady state is given, in usual notation, as

$$x = \frac{P}{m\omega_n^2} \frac{1}{\sqrt{(1-r^2)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi) \text{ where } r = \frac{\omega}{\omega_n}, \quad \omega_n = \sqrt{\frac{k}{m}} \text{ and } \phi = \tan^{-1}\left(\frac{2\beta r}{1-r^2}\right).$$

(a) Show that the isolation factor (i) is given by

$$i = \left[\frac{1+(2\beta r)^2}{(1-r^2)^2 + (2\beta r)^2}\right]^{1/2}$$

(b) Show that for effective isolation ($i < 1$), the frequency ratio, $r > \sqrt{2}$.

(c) An industrial sewing machine has a mass of 450 kg and operates at 1500 rev/min. It has a rotating unbalance of magnitude 0.75 kgm and it is isolated from a rigid foundation by an isolator. What should be the stiffness of an undamped isolator which will limit the force transmitted to the foundation to 9500 N.

(d) If a critically damped isolator is used instead of the undamped isolator what should be the stiffness of the isolator.

QUESTION 02

Fig. Q2 shows pulleys, A and B of radii r_1 and r_2 and polar moments of inertia I_1 and I_2 respectively. These pulleys can rotate about their polar axes. One end of a string stretched over pulley A is connected to a fixed spring of stiffness k_1, while its other end is attached to a second spring of stiffness k_2. The free end of the second spring is connected to a second string and the free end of the second string is connected to a third spring of stiffness k_3. The free end of the third spring carries a mass m. Assume that the strings do not slip on the pulleys. Determine the equations of motion for small amplitude of oscillations.

If $r_1 = r$, $r_2 = 2r$, $I_1 = 2mr^2$, $I_2 = 6mr^2$, $k_1 = k_2 = k_3 = k$, show that the frequency equation can be expressed as

$$(\alpha - 1)(3\alpha^2 - 7\alpha + 1) = 0$$

where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of oscillations.

Determine the natural frequencies in terms of k and m.

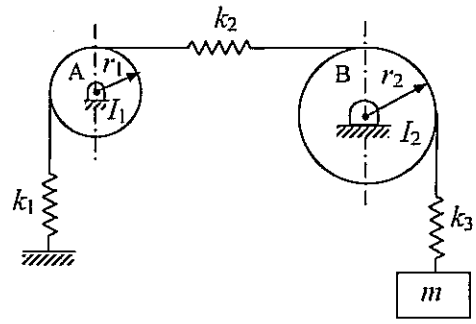


Fig. Q2

QUESTION 03

Fig. Q3 shows a torsional system in which three rotors of moments of inertia I_1, I_2 and I_3 are connected by shaft sections having torsional stiffnesses K_1, K_2, K_3 and K_4. Determine the equations of motion and obtain the frequency equation for small torsional oscillations.

Show that if $K_1 = K_2 = K$, $K_3 = K_4 = 3K$, $I_1 = I$, $I_2 = 2I$ and $I_3 = 3I$, the frequency equation can be expressed as $(\alpha - 2)(\alpha^2 - 4\alpha + 2) = 0$ where $\alpha = \dfrac{I\omega^2}{K}$ and ω is the natural frequency of torsional oscillations.

Determine the natural frequencies in terms of I and K and the mode shapes.

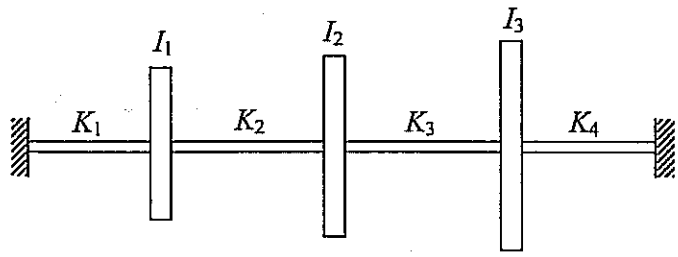


Fig. Q3

QUESTION 04

Starting from fundamentals, obtain an expression for the fundamental natural frequency of transverse vibration of a cantilever of length L and mass per unit length m using Energy method. Assume that the vibration deflection curve is of the same form as the static deflection curve.

Hence determine the frequency of transverse vibration of a steel turbine blade of uniform section and of length 150 mm, having a mass of 2 kg/m and a moment of inertia of 2600 mm^4. Take the modulus of elasticity of steel as 200 GN/m^2.

QUESTION 05

(a) For the system shown in Fig. Q5(a), determine the range of values of K such that the system is stable. Write the factored form of the characteristic equation for the case in which the characteristic equation has a pair of complex conjugate roots on the imaginary axis.

Also determine the unit impulse response of the system for $K = 10$. Assume all initial conditions to be zero

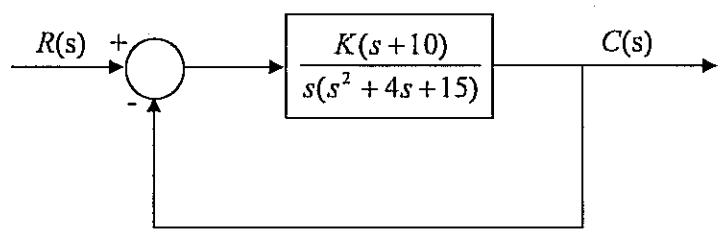


Fig. Q5(a)

(ii) Fig. Q5(b) shows a closed loop control system with tachometer feedback. What is the value of K for the system to have a damping ratio of 0.75?

Also determine the peak time, percentage overshoot and the settling time of the control system for the value of K thus determined.

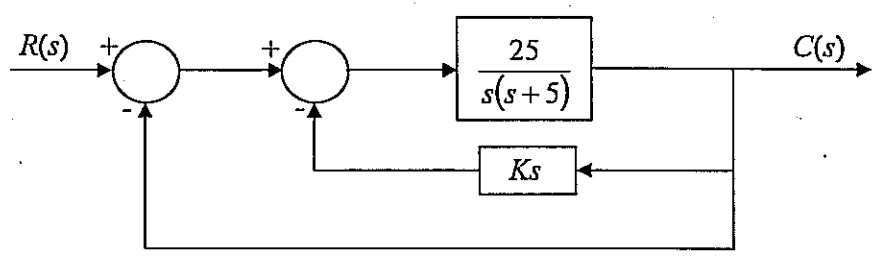


Fig. Q5(b)

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K(s+6)}{(s+1)(s^2+6s+13)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and comment on the stability of the closed loop system.

(ii) Using the root locus you have drawn show that at $s = -5$ the gain $K = 32$. Determine the other closed loop poles for $K = 32$.

(iii) Determine the damping ratio of the closed loop system when $K = 32$.

(iv) Determine the unit impulse response of this system for $K = 32$. Assume that all initial conditions are zero.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s)=\frac{4}{s(1+0.5s)(1+0.08s)}.$$

Draw a Bode plot and determine the gain margin and the phase margin of the closed loop control system.

Also determine the factor by which the gain has to be changed in order to have a gain margin of 7 db.

QUESTION 08

Fig. Q8 shows a hydraulic servomechanism in which the "walking beam" connects the input position x, the valve position z and the piston position y. One end of the piston rod of the piston in the working cylinder is connected to a mass m which in turn is connected to a spring and a dashpot. The spring has a stiffness k and the damping constant of the dashpot is c. When z is zero, the valve is fully closed and the walking beam is vertical. The rate of flow of oil to and from the working cylinder, of which the piston has an area of cross section A, is q_1 and it is given by,

$q_1 = B_1 z$, where B_1 is a constant.

The leakage past the piston in the working cylinder is q_2 and it is given by

$q_2 = B_2 p$, where B_2 is a constant and p is the pressure difference across the piston..

Draw a complete block diagram of the overall system and obtain the transfer function relating x and y.

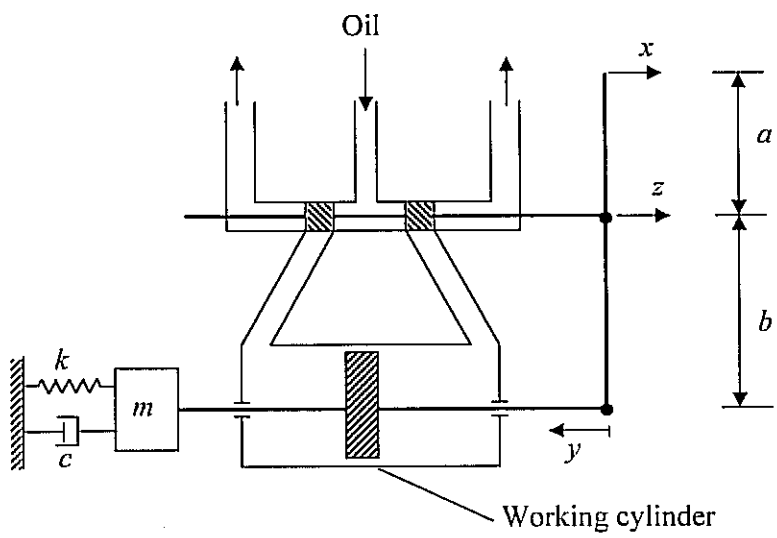


Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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