THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
BACHELOR OF TECHNOLOGY – LEVEL 05
FINAL EXAMINATION– 2011/2012

MEX5230 – FLUID MECHANICS

DATE : 21st MARCH 2012
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE HOURS

ANSWER FIVE QUESTIONS ONLY. ALL QUESTIONS CARRY EQUAL MARKS.

**Unless specified, assume density of water = 1000 kg/m$^3$. Acceleration due to gravity= 10m/s$^2$.**

QUESTION 1

a) Find horizontal and vertical components and the resultant of the hydrostatic thrust exerted by water on one meter length of the cylinder shown in Figure Q1. Also determine the direction of this resultant which makes with the horizontal.

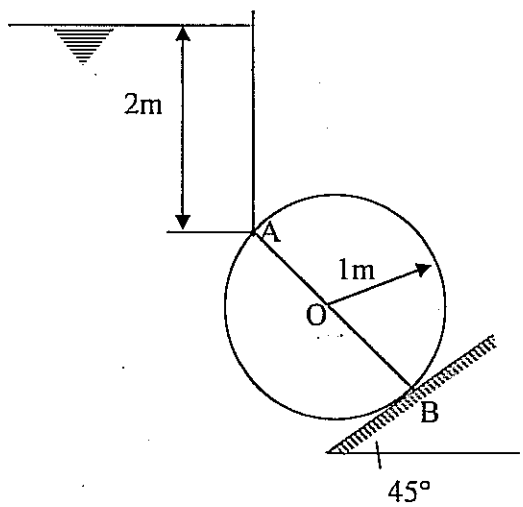



Figure Q1

b) The actual velocity in the contracted section of a jet of liquid ejecting from a 50 mm diameter orifice is 7.5 m/s under a constant head of 4m. Calculate the coefficient of velocity. If the measured discharge is 0.008 m$^3$/s, determine the coefficient of contraction and coefficient of discharge for this orifice.

## QUESTION 2

The non-dimensional factors, which are adopted to analyse centrifugal pump performances by the principle of similarity, is given by

$$\frac{H}{N^2D^2}, \frac{Q}{ND^3}, \frac{P}{N^3D^5}$$

Where $Q$ is the discharge, $N$ is rotational speed of the pump, $D$ is diameter of the runner and $P$ is power developed by the pump.

To predict the performance of a large centrifugal pump, a scale model of one fifth size was made with the following specifications.

P = 30kW

Head (H) = 8m

N = 1000rev/min

If the larger pump has to work against a head of 25m, make calculations for its working speed, the power required to drive it and the ratio of the flow rates handled by the two pumps.

## QUESTION 3

State Buckingham's Pi theorem used in dimensional analysis

Using Buckingham's Pi theorem show that the velocity ($V$) of a liquid through an orifice can be written as

$$V = \sqrt{gH}\oint\left(\frac{D}{H}, \frac{\mu}{\rho VH}, \frac{\sigma}{\rho V^2H}\right)$$

where $H$ is the head of liquid above the orifice, $D$ is the diameter of the orifice, $\mu$ is the coefficient of dynamic viscosity, $\rho$ is the mass density, $\sigma$ is the surface tension and $g$ is the acceleration due to gravity.

## QUESTION 4

An incompressible fluid with dynamic viscosity $\mu$ flows in a horizontal circular pipe of radius R when a longitudinal pressure gradient $\frac{dp}{dx}$ exists.

a) If the flow is steady, fully developed and laminar,

i. Show that the momentum equation governing fluid motion is

$$r\frac{dp}{dx} = \frac{d}{dr}(r\tau)$$

where $\tau$ is the shear stress in the fluid.

It is also given that $\tau = \mu\frac{du}{dy}$, where $u$ is the fluid velocity at radius $r$.

ii. Hence show that the fluid velocity in the pipe is given by

$$\frac{u}{u_0} = 1 - \left(\frac{r}{R}\right)^2$$

where $u_0$ is the fluid velocity along the centre line of the pipe.

b) Show that;

i. the average fluid velocity in the pipe is half of the centerline velocity.

ii. the pressure gradient in the flow is given by

$$\frac{dp}{dx} = -\frac{8\mu\dot{m}}{\pi\rho R^4}$$

where $\dot{m}$ is the fluid mass flow rate and $\rho$ is the fluid density.

iii. In a particular pipe flow, R=0.1m, $\mu = 0.104Ns/m^2$, $\rho = 870kg/m^3$ and the pressure gradient $\frac{dp}{dx} = -50N/m^3$.

Calculate;

i) the fluid mass flow rate

ii) the maximum fluid velocity

## QUESTION 5

Two reservoirs 10km apart have a difference of water level of 25 m and are connected together by a pipeline. The pipeline consists of 6 km single pipe feeding a junction from which two pipes run in parallel for the remainder length of 4 km to the lower reservoir. If the same inner diameter of pipe is used throughout, what should be the diameter of the pipes so that velocity may not exceed 1.25m/s. Take the friction factor 4f = 0.03.

a) Find the diameter of the equivalent pipe with 4000m long.
b) Find the length of the equivalent pipe with 450mm in diameter.
c) If the three pipes are connected in parallel, find the equivalent length of a 600mm diameter pipe.

**QUESTION 6**

a. Write down one of the common forms of the Bernoulli equation applied to an incompressible fluid flowing through a control volume and state the conditions under which its application is valid.

b. Figure Q6 shows a converging elbow which turns a flow of water through an angle of 135° in vertical plane. The internal diameters of the elbow at inlet section (1) and outlet section (2) are 400 mm and 200 mm respectively. The water volume in the elbow between sections (1) and (2) is 0. 2 $m^3$ and the elbow has a mass of 12 kg. The steady water volume flow rate through the elbow is 0.4 $m^3/s$ when the gauge pressure at section (1) is 150kN/$m^2$. Assuming losses in the flow through the elbow are negligible, calculate:

i) The water velocity and pressure at the outlet section (2).

ii) The magnitude and direction of the resultant force acting on the flow.

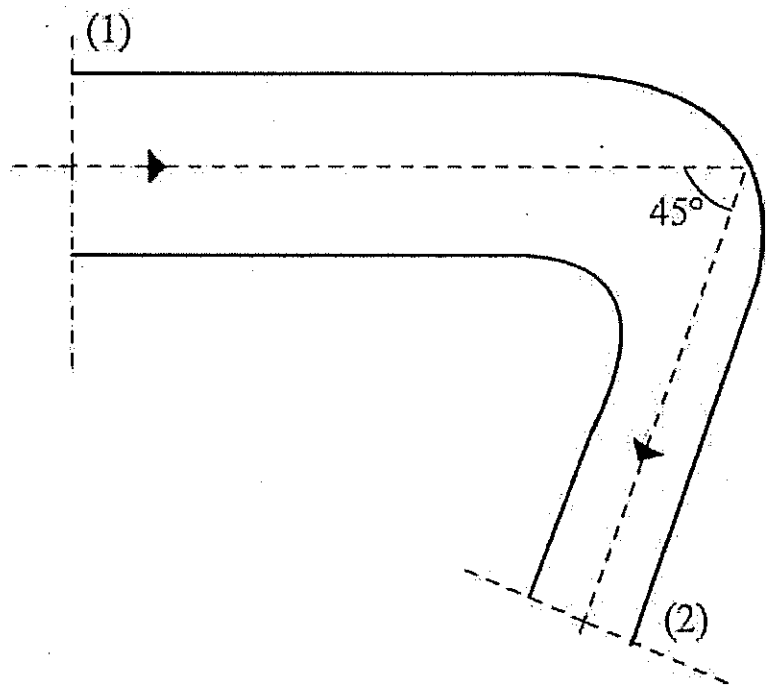



Figure Q6

## QUESTION 7

Laminar flow over a thin flat plate causes a boundary layer to develop, starting at the leading edge of the plate and growing thickness in the direction x along the plate.

a. The velocity distribution in the boundary layer is assumed to be of the form

$$\frac{u}{U} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^3$$

In which $\boldsymbol{U}$ is the free stream velocity, $\boldsymbol{u}$ is the velocity in the boundary layer at distance y from the plate surface, $\delta$ is the boundary layer thickness and $\boldsymbol{a, b}$ and $\boldsymbol{c}$ are constants. Using suitable boundary conditions show that

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

b. Also, show that the displacement thickness($\delta^*$) and the momentum thickness ($\theta$) for the boundary layer with the assumed velocity profile given in part a, are $0.375\delta$ and $0.1393\delta$ respectively.

c. Use the momentum integral equation $\frac{d\theta}{dx} = \frac{\tau_0}{\rho U^2}$, to show that the boundary layer thickness $\boldsymbol{\delta}$ at distance x along the plate is given by

$$\frac{\delta}{x} = \frac{4.641}{\sqrt{Re_x}}$$

where $\tau_0$ is the viscous shear stress at the plate surface

## QUESTION 8

Two sources each of strength $m$ are placed at the points (-a,0),(a,0) and a sink of strength $2m$ at the origin. Show that the stream function is given by

$$\left(x^2 + y^2\right)^2 = a^2\left(x^2 - y^2 + \lambda xy\right)$$

where $\lambda$ is a variable parameter for a given stream line.

Also, show that the speed at any point is $\frac{2ma^2}{(r_1 r_2 r_3)}$, where $r_1, r_2$ and $r_3$ are the distances of the point from the two sources and the sink respectively. You may use the following relationship.

$$z = (x + iy) \text{ and } \ln(\alpha + i\beta) = \frac{1}{2}\ln\left(\alpha^2 + \beta^2\right) + i\tan^{-1}(\beta/\alpha)$$

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