THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Level 05)

MPZ 5230-Engineering Mathematics (III)

Final Examination (2012/2013)

Duration: Three (03) hours

Date: 30/07/2013 Time: 09.30-12.30

Instructions:

- Answer **all** questions
- This is a **closed book** examination
- This paper contains **04-pages**
- Show all your workings
- All symbols are in standard notations

1.

a) Let $\underline{\boldsymbol{H}} = (6xy + z^3)\boldsymbol{i} + (3x^2 - z)\boldsymbol{j} + (3xz^2 - y)\boldsymbol{k}$.

i. Evaluate $Curl\ \underline{\boldsymbol{H}}$.

ii. Find φ such that $\underline{\boldsymbol{H}} = grad\varphi$. [40 %]

b) Elliptic cylinder coordinates (u, v, z) are defined by the equations $x = \lambda\ coshu\ cosv, y = \lambda\ sinhu\ sinv$ and $z = z$, where λ is a constant and $u \geq 0,\ 0 \leq v \leq 2\pi$.

i. Obtain the scale factors in this coordinate system.

ii. Express Laplace's equation $div(grad\emptyset) = 0$ in this coordinate system.

[60 %]

2.

a) Using polar coordinates, evaluate

$$\iint_A (x+y)^2 dxdy,$$

where $A = \{(x,y): 1 \leq x^2 + y^2 \leq 4\}$. [20 %]

b) Use Green's theorem to evaluate

$$\iint_S (x^2 + y^2) dxdy,$$

where $S = \{(x,y): x^2 + y^2 \leq 1\}$. [30 %]

c) Evaluate

$$\iint_R (3x + 6y)^2 dxdy,$$

where R is the region bounded by the coordinates $A(0,1)$, $B(2,0)$, $C(0,-1)$ and $D(-2,0)$. [30 %]

d) Let $V = \{(x,y,z): x^2 + y^2 + z^2 \leq 1\}$. Using spherical polar coordinates evaluate

$$\iiint_V \sqrt{x^2 + y^2 + z^2}\, dxdydz.$$ [20 %]

3.

a) Find the region of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(Z+1)^n}{(n+2)^2} \left(\frac{1}{3}\right)^n.$$ [30 %]

b)

i. Find the Taylor series representation of the function $\frac{1}{1-Z}$ in $|Z + i| < \sqrt{2}$. [20 %]

ii. Find the Laurent series representation of the function $exp\left(\frac{-1}{z}\right)$ in $0 < |Z| < \infty$. [20 %]

c) Apply residue theorem to evaluate

$$\int_C \frac{e^z}{Z(Z+2)} dZ,$$

where C is represented by $|Z-1| = 5$. [30 %]

4.

a) Find the Fourier transform of $f(x) = \begin{cases} 1\,;\ 0 \le x < 1 \\ x\,;\ 1 \le x < 2 \\ 0\,;\ otherwise \end{cases}$. [15 %]

b) Find the Fourier cosine transform of $h(x) = \left(1 - \frac{x}{\pi}\right); 0 < x < \pi$. [15 %]

c) Using the variables separable method, solve

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}\ ; 0 < x < a, t > 0,$$

subject to $U(0,t) = U(a,t) = 0$ and $U(x,0) = x(a-x)$. [70 %]

5.

Consider the second order symmetric tensor whose components are given by:

$$[T_{ij}] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

i. Find the eigen values and the principle axes of the tensor.

ii. Express the tensor with respect to its principle axes.

iii. Write the equation of the transformation relating to the coordinates X_i determined by the principal axes.

iv. Verify the result in (ii) by using the transformation law for second order tensor.

[100 %]

6.

a) An investor has Rs.600,000 to allocate among following investment alternatives:

Investment alternatives	Rate of return
Fixed income funds	7.5 %
Equity income funds	8.6 %
Aggressive growth funds	13.5 %
Balanced funds	7.8 %

The investor has the following investment criteria:

- The full amount of Rs.600,000 is to be invested.
- At least Rs.100,000 is to be invested in fixed income funds.
- At least Rs.100,000 is to be invested in equity income funds.
- At least Rs.60,000 is to be invested in aggressive growth funds.
- At least Rs.50,000 is to be invested in balanced funds.

Formulate (do not solve) a linear programming model to determine the amount that should be invested in each investment alternatives in order to maximize the annual return.

[30 %]

b) Consider the primal linear programming problem:

$$\begin{aligned} Maximize\ Z = x_1 + x_2 + 3x_3 \\ subject\ to \quad 3x_1 + 2x_2 + x_3 \leq 3 \\ 2x_1 + x_2 + 2x_3 \leq 2 \\ x_1, x_2, x_3 \geq 0. \end{aligned}$$

i. Use simplex method to solve this primal linear programming problem.

ii. Using the optimal table in (i), obtain the solution for dual problem.

iii. Verify the result in (ii), by applying graphical method for dual problem.

[70 %]

END