THE OPEN UNIVERSITY OF SRI LANKA

FACULTY OF ENGINEERING TECHNOLOGY

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2006/2007

MEX3273 – MODELLING OF MECHATRONIC SYSTEMS

049

DATE : *20 MARCH 2007*

TIME : *0930HRS -1230HRS*

DURATION : *THREE HOURS*

PLEASE READ THE FOLLOWING INSTRUCTIONS BEFORE ANSWERING THE PAPER

INSTRUCTIONS:

1. *This paper is of closed book type.*
2. *This paper consists of eight questions.*
3. *You are required to answer any **five questions only**.*
4. *Answers should be written on the answer books and sheets provided by the Examinations Division.*

Question 01

(1.1) Explain the terms 'System' and 'Model' as applied in Systems Engineering. Elaborate with suitable examples.

(1.2) Briefly explain the procedure carried out when developing a model for a particular system.

(1.3) Discuss the potential benefits and drawbacks of system modeling.

(1.4) 'Mechatronics products offer superior performance over their counterparts by adopting an integrated design approach' Comment on the statement.

Question 02

(2.1) Distinguish between a 'lumped parameter system' and a 'distributed parameter system' in the context of modeling.

(2.2) Show that a heavy spring of mass 'M' and a stiffness of 'K' can be approximated to a massless spring of the same stiffness and a discrete mass of M/3 located at its end.

(2.3) Explain the following properties in relation to systems.

(a) Causality

(b) Time invariance

(c) Linearity

(d) Memoryless

(2.4) The input-output relationship of a system is given by, $y = \mathrm{T}\{x\} = x^2$. Determine whether the system is,

(a) Linear

(b) Time-invariant

(c) Causal

(3.2

Question 03

(3.1) Using the force-voltage analogy, show that springs connected parallel is analogous to electrical capacitors connected in a similar fashion.

(3.2) For the spring-mass-pulley system shown in figure 3.1, find the mathematical model relating the motion of the system. Displacement x is measured from its equilibrium position, and take moment of the pulley about its axis of rotation as J and radius R.

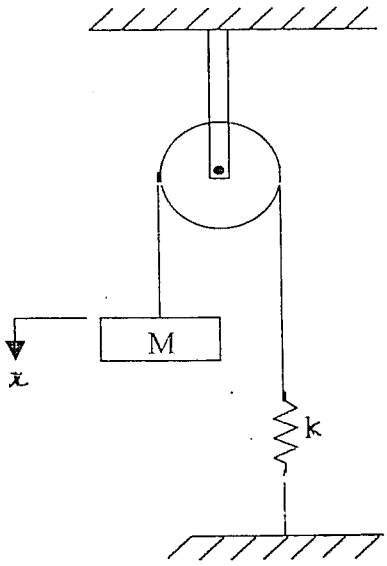


Figure 3.1

(3.2) Figure 3.2 represents an inverted pendulum mounted on a motor driven cart, which is often used to model the attitude control of a space booster during its takeoff. A control force *u(t)* is applied to the cart in order to prevent the pendulum from falling over.

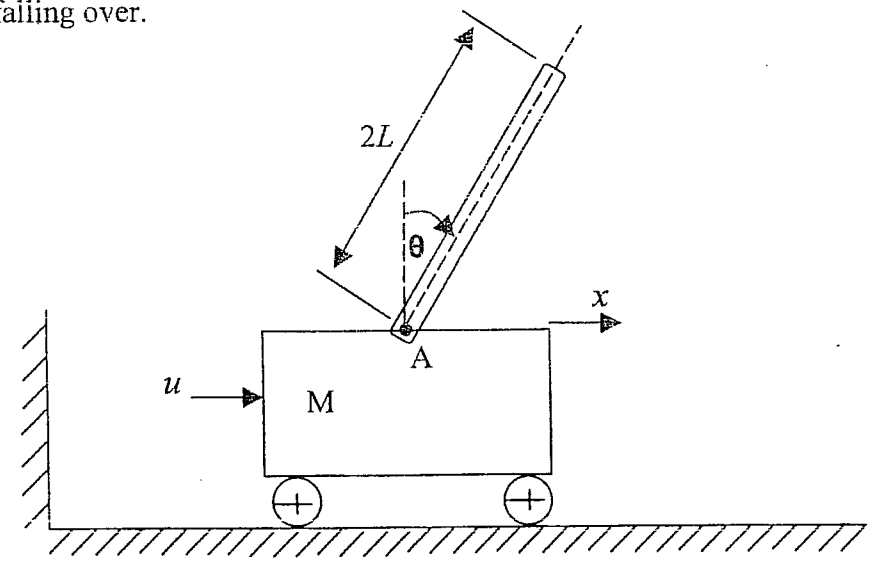


Figure 3.2

The masses of pendulum and cart are m and M respectively. Assuming that the center of gravity of the pendulum rod is at its geometrical center, and no frictional forces act between the contact surfaces between the cart and floor,

(a) Obtain the mathematical model for the system.

(b) Linearize the model for small rotational angles of the pendulum.

Question 04

(4.1) Consider the electrical system shown in figure 4.1 below.

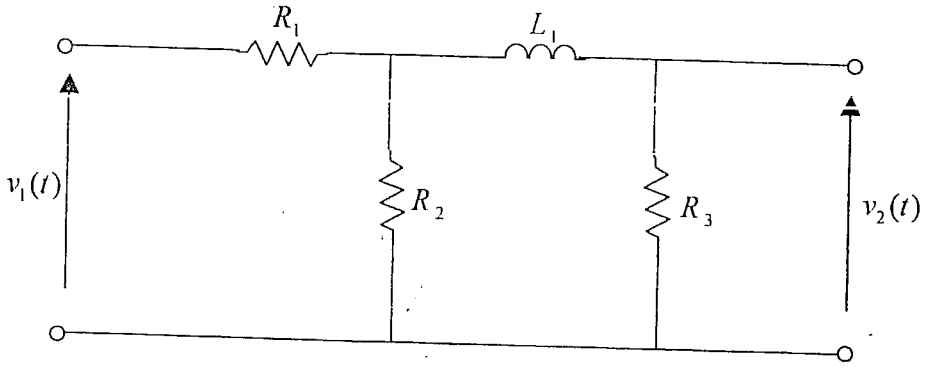


Figure 4.1

(a) Find the voltage transfer function, V_2/V_1

(b) Determine the voltage transfer function, V_2/V_1, if an indictor L_2 is connected across the output terminals parallel with the resistor R_3.

(4.2) Figure 4.2 shows a spring-mass-dashpot system mounted on a massless cart. An input of *u(t)* is given to the cart and the displacement of the mass *y(t)* is measured.

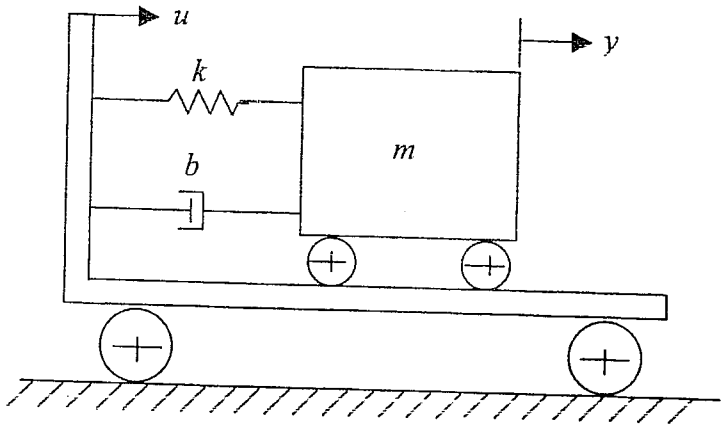


Figure 4.2

(a) Find the transfer function, which relates the input and output, considering the friction between the mass and cart, and the cart and floor to be negligible.

(b) If m =10 kg, b = 20 N-s/m and k = 100 N/m, determine the output *y(t)*, when an unit step function is given to the system.

Question 05

An equivalent circuit of a loudspeaker is shown in figure 5 below. R, L and C are the resistance, inductance and capacitance of the loudspeaker coil while m, k and b are the mass, stiffness and the friction in the loudspeaker cone and its accessories. Determine the transfer function of the loudspeaker, when the input voltage is *v(t)* and the corresponding cone displacement is *x(t)*. State all assumptions you make.

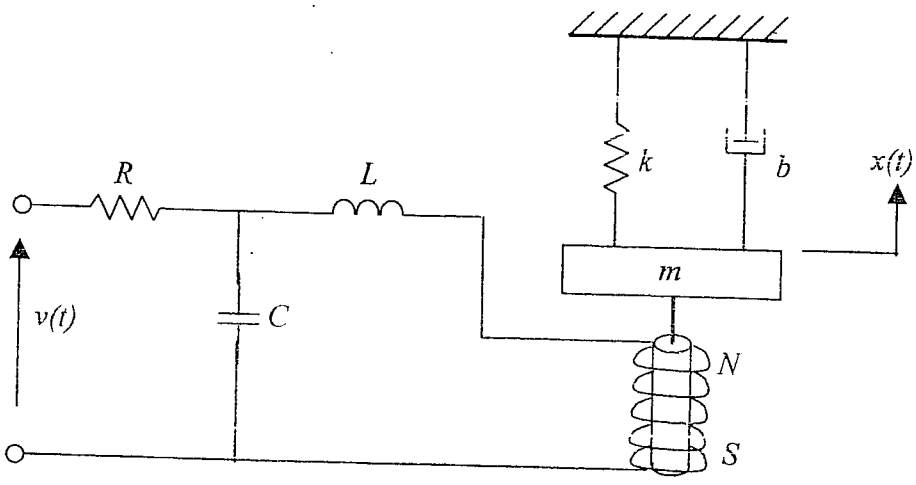


Figure 05

Question 06

(6.1) Consider the translational mechanical system shown in figure 6.1.

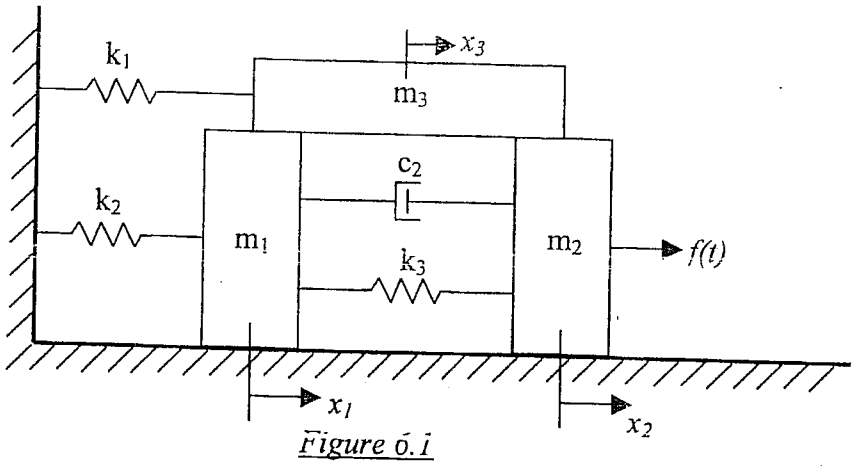


Figure 6.1

$m_1 = m_2 = m_3 = 1$ kg ; $k_1 = k_2 = k_3 = 1$ N/m ; $c_2 = 1$ N-s/m

The friction between m_1 and floor, and m_2 and floor is negligible.

The friction between m_3 and m_1, m_3 and m_2 are c_1 and c_3 respectively.

$c_1 = c_3 = 1$ N-s/m

By taking the above values into consideration,

(a) Draw free body diagrams (FBD) for the forces acting on masses m_1, m_2 and m_3.

(b) Using impedance method, write the governing equations for the system.

(6.2) Using impedance method, find the transfer function V_0 / V_i for the electrical circuit shown in figure 6.2.

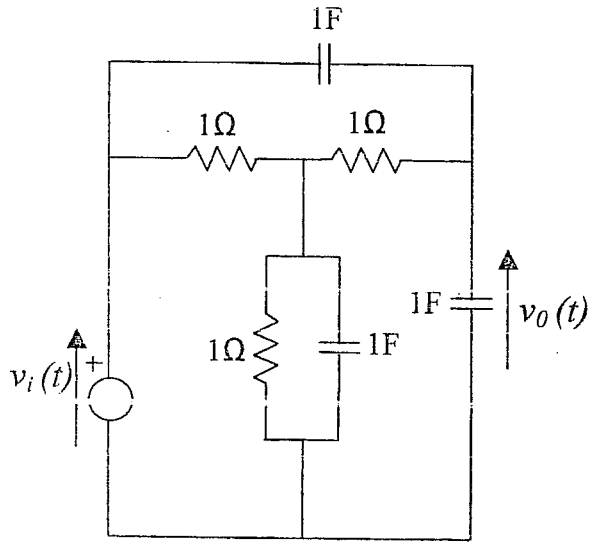


Figure 6.2

Question 07

(7.1) Define the following terms with respect to state-space representation.

(a) State of a system

(b) State variables

(c) State vector

(d) State-space

(7.2) Represent the transnational mechanical system given in figure 7 in state space form. You may take $f(t)$ and x_3 to be the input and output respectively.

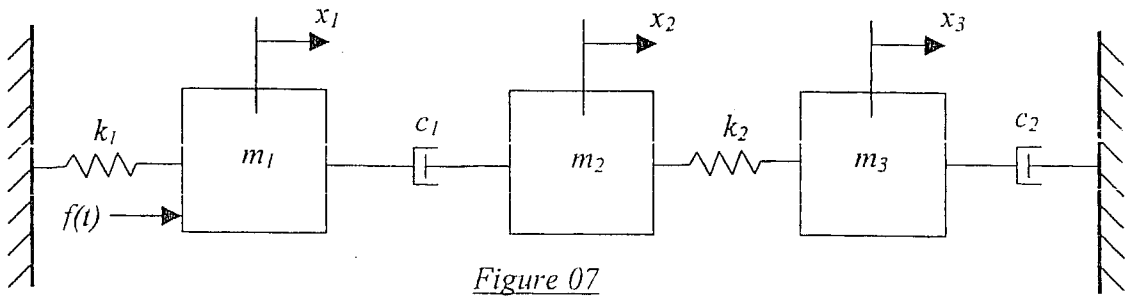


Figure 07

In figure 07,

$m_1 = m_2 = m_3 = 1$ kg
$c_1 = c_2 = 1$ N-s/m
$k_1 = k_2 = 1$ N/m

(7.3) Using the phase-variable method, find the state space form of the transfer function given below.

$$\frac{C(s)}{R(s)} = \frac{30}{s^3 + 10s^2 + 10s + 35}$$

Question 08

(8.1) Briefly explain the functional elements of;

(a) Block diagrams

(b) Signal flow graphs

(8.2) Reduce the block diagram given in figure 08 to open loop form. You are required to show important steps in answering the question 8.2.

(8.3) Develop a signal flow graph for the block diagram in figure 08.

(8.4) Obtain the transfer function for the signal flow graph in question (8.3) using Mason's Gain Formula.

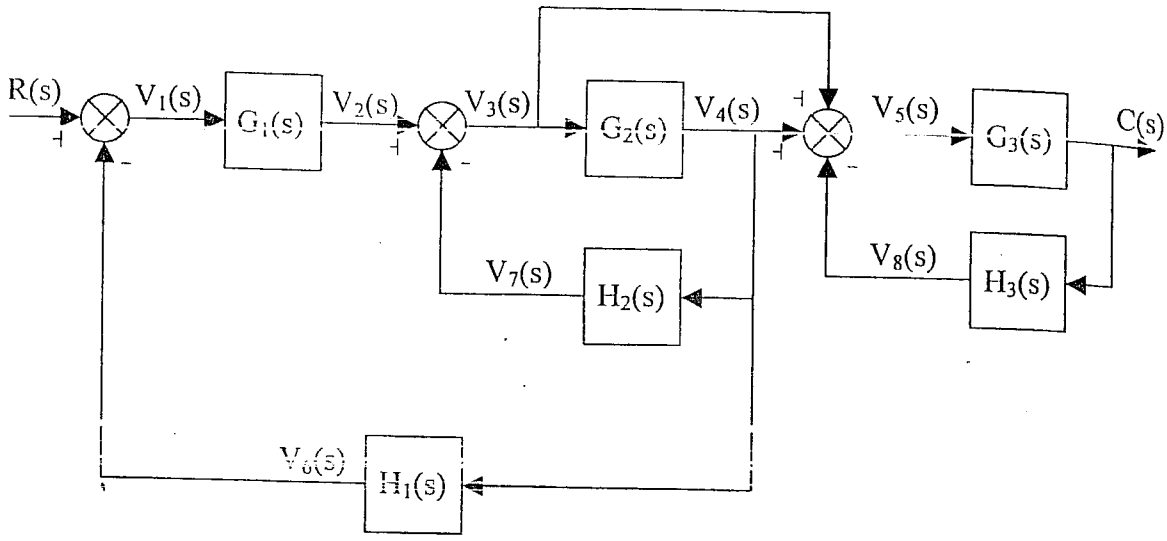


Figure 08

Mason's Gain Formula:

$$G(s)=\frac{C(s)}{R(s)}=\frac{\sum_k T_k \Delta_k}{\Delta}$$ Where,

k = number of forward paths

T_k = the k^{th} forward path gain

Δ = 1 – Σ loop gains + Σ non-touching loop gains taken two at a time – Σ non-touching loop gains taken three at a time + Σ non-touching loop gains taken four at a time - .....

Δ_k = Δ - Σ loop gain terms in Δ that touch the k^{th} forward path. That is, Δ_k is formed by eliminating from Δ those loop gains that touch the k^{th} forward path.

~ END ~

ALL RIGHTS RESERVED

LAPLACE TRANSFORM TABLE

f(t)	F(s)
Unit impulse $\delta(t)$	1
Unit step $u(t)$	$1/s$
t	$1/s^2$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$