THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
DIPLOMA IN TECHNOLOGY – LEVEL 4
FINAL EXAMINATION – 2009

MEX4272 – VIBRATION AND FAULT DIAGNOSIS

DATE : 05 th MARCH 2009
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

Fig. Q1 shows a rigid uniform rod of mass $3m$ and length $4a$, pivoted at one end and carrying a mass $2m$ at its mid point. A spring of stiffness $2k$ is attached to the rod at a distance a below the pivot while another spring of stiffness k is attached at the lower end of the rod as shown. Determine the equation of motion and show that the natural frequency of vibration of the system (ω_n) is given by

$$\omega_n = \sqrt{\frac{3}{4}\frac{k}{m} + \frac{5}{12}\frac{g}{a}}$$

Also calculate the natural frequency of vibration of the system if the mass $2m$ and the spring of stiffness $2k$ are removed from the system.

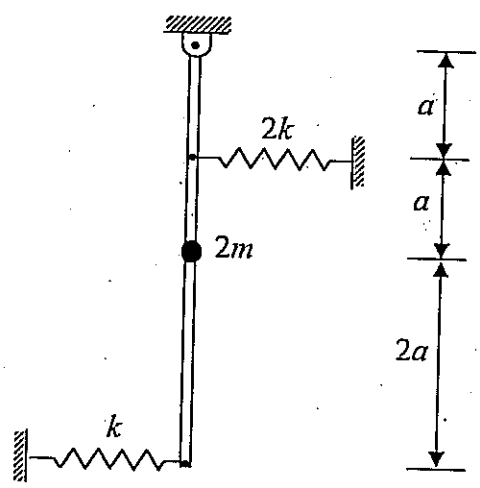


Fig. Q1

QUESTION 02

Fig. Q2 shows a spring-mass-damper system where the mass m is supported on a spring of stiffness k and a damper of damping constant b. Determine the equation of motion and show that it can be written as

$$\ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x = 0$$

where $\zeta = \dfrac{b}{2\sqrt{km}}$ and $\omega_n = \sqrt{\dfrac{k}{m}}$

Derive expressions for the free response (motion of the mass denoted by x) of the system for the two cases, (i) damping factor $\zeta < 1$ (ii) damping factor $\zeta = 1$, and sketch the approximate shape of the response curves.

If, m = 3 kg, b = 90 Ns/m, k = 375 N/m and the initial conditions are $x(0) = 1$, $\dot{x}(0) = 1$, obtain the free response of the system.

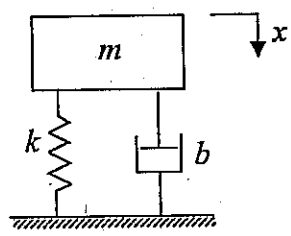


Fig. Q2

QUESTION 03

Fig. Q3 shows a massless rigid rod of length $2L$, pivoted at its mid point and carrying a mass $2m$ at each end. Three springs each of stiffness k are attached to the masses as shown. The system of masses and springs is constrained to move only in the vertical direction. Determine the natural frequencies of oscillations of the system and the corresponding modes.

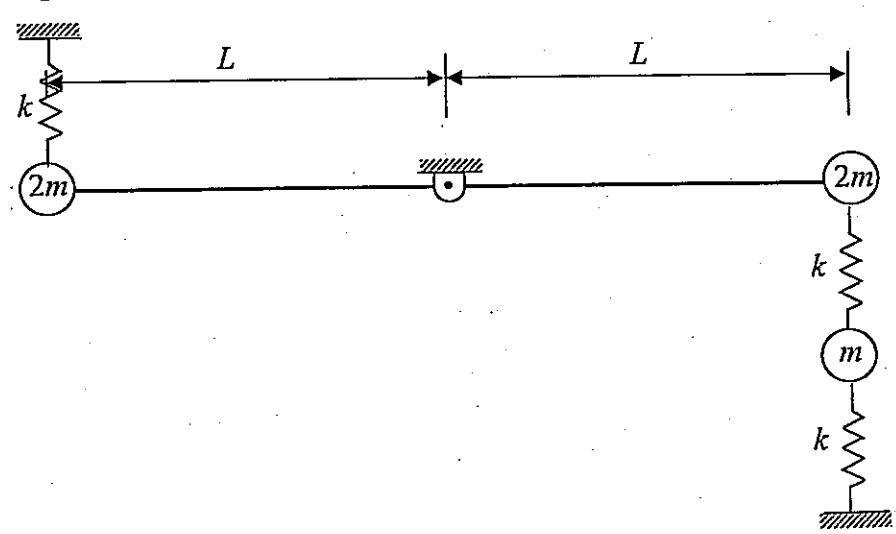


Fig. Q3

QUESTION 04

Fig. Q4 shows a spring-mass system consisting of two masses and four springs. The parameters of the masses and springs are indicated in the figure. Determine the equations of motion and obtain the frequency equation.

If $m_1 = m$, $m_2 = 2m$, $k_1 = 2k$ and $k_2 = k_3 = k$, determine the natural frequencies of vibration of the system and modes. Sketch the relevant mode shapes.

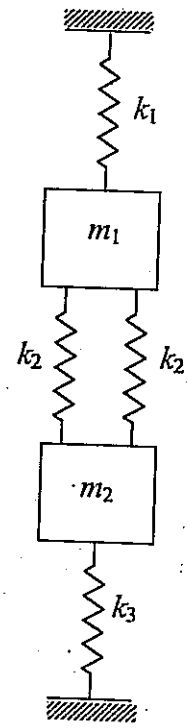


Fig. Q4

QUESTION 05

Fig. Q5 shows a two degree of freedom system. It consists of two rotors of momemts of inertia I_1 and I_2 and three shaft sections having torsional stiffnesses K_1, K_2 and K_3. Determine the equations of motion and obtain the frequency equation.

If $K_1 = K_2 = K$ Nm/rad , $K_3 = 2K$ Nm/rad, $I_1 = 4I$ kg m^2 and $I_2 = I$ kg m^2, determine the natural frequencies of torsional oscillations and the mode shapes.

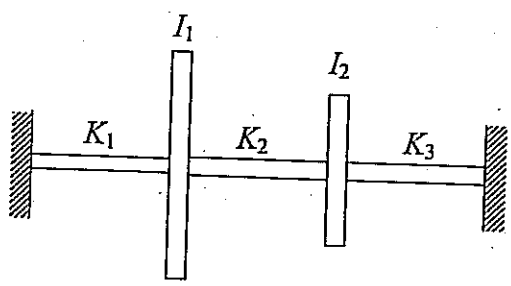


Fig. Q5

QUESTION 06

A mechanical system that is at rest is subjected to a unit step input $u(t)$. Its response is given by $y(t) = 1 - e^{-t}\cos 2t - 0.5\, e^{-t}\sin 2t$. Determine the following.

(i) Transfer function of the system.

(ii) Input-output differential equation of the system.

(iii) Damped natural frequency, undamped natural frequency and the damping ratio.

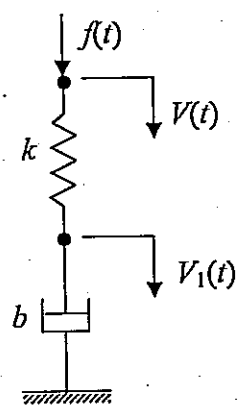
(iv) Response of the system to a unit impulse input.

(v) Steady state response for a unit step input?

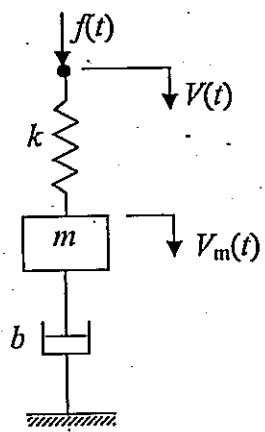
QUESTION 07

(i) Determine the mechanical impedance and mobility of the following systems.

(a)



(b)



(ii) Obtain the time response of the dynamic system described by the differential equation, $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 5\frac{dx}{dt} + 5x$, where x (input) $= u(t)$ (Unit step input).
Assume all initial conditions to be zero.

QUESTION 08

(i) Mechanical vibrations are known to have harmful effects as well as useful ones. Briefly describe three practical examples each for good vibration and bad vibration.

(ii) Write short notes on the following.

(a) Measurement accuracy

(b) Instrument accuracy

(c) Precision.

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-.....-f^{n-1}(0)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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