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**THE OPEN UNIVERSITY OF SRI LANKA**
**FACULTY OF ENGINEERING TECHNOLOGY**
**DIPLOMA IN TECHNOLOGY – LEVEL 4**
**FINAL EXAMINATION – 2010**

**MEX4272 – VIBRATION AND FAULT DIAGNOSIS**

**DATE** : **04th MARCH 2011**
**TIME** : **0930 HRS. – 1230 HRS.**
**DURATION** : **THREE (03) HOURS**

**ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS**

**QUESTION 01**

A uniform slender rod of mass $m$ and length $L$ is pivoted at point A and is attached to four linear springs each of stiffness $k$ and a torsional spring of stiffness $k_t$ as shown in Fig. Q1. Derive an expression for the natural frequency ($\omega_{no}$) for small oscillations of the system.

Show that if $k_t = 0$, the natural frequency ($\omega_{n1}$) of oscillations is given by

$$\omega_{n1} = \sqrt{10\frac{k}{m} + \frac{3}{2}\frac{g}{L}}$$

Evaluate the natural frequency ($\omega_{no}$) if $k = 2000$ N/m, $k_t = 1000$ Nm/rad, $m = 10$ kg and $L = 5$ m.

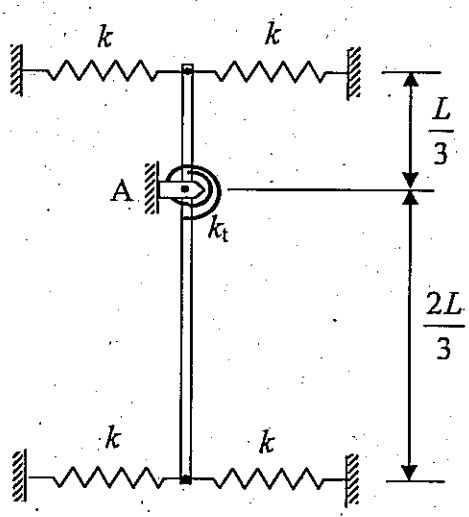



Fig. Q1

**QUESTION 02**

Fig. Q2 shows a spring-mass-damper system in which the movement of the mass $m$ is restrained by three springs of stiffnesses $k_1$, $k_2$ & $k_3$ and a damper of damping constant $b$. Determine the equation of motion.

If $m$ = 5 kg, $k_1$ = 1840 N/m, $k_2$ = 200 N/m, $k_3$ = 800 N/m and $b$ = 100 Ns/m, determine the natural frequency and damping ratio of the system. Determine the free response of the system assuming that the initial conditions are : $x(0) = 0$ and $\dot{x}(0) = 10$ mm/s.

If the damper is now replaced by a new damper which is critically damped what would be the free response of the system? Assume that the initial conditions are, $x(0) = 10$ mm and $\dot{x}(0) = 10$ mm/s.

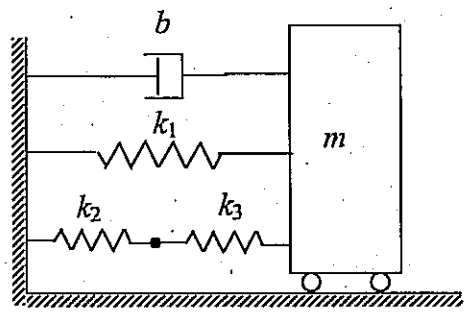



Fig. Q2

**QUESTION 03**

Fig. Q3 shows two slender rods, each of mass $m$ and length $L$, pivoted at one of its ends. The rods are connected at their mid points by a spring of stiffness $k$. Each rod carries a mass $m$ attached to its free end. Determine the equations of motion for small oscillations and show that the frequency equation can be expressed in the form,

$$\left(\omega^2 - \alpha\frac{g}{L} - \beta\frac{k}{m}\right)^2 - \left(\beta\frac{k}{m}\right)^2 = 0,$$

where $\omega$ is the natural frequency of oscillations and $\alpha$ & $\beta$ are numerical constants. What are the values of $\alpha$ and $\beta$.

If $m$ = 2 kg, $L$ = 1 m, $k$ = 1000 N/m and $g$ = 10 m/s$^2$, determine the natural frequencies.

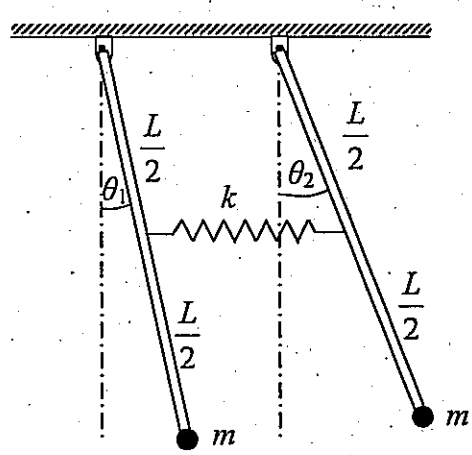



Fig. Q3

**QUESTION 04**

Fig. Q4 shows a spring-mass system consisting of two masses $m_1$ & $m_2$ and three springs of stiffnesses $k_1$, $k_2$ & $k_3$. Determine the equations of motion of the system and obtain the frequency equation.

If $m_1 = m$, $m_2 = 2m$, $k_1 = k$ and $k_2 = 2k$ and $k_3 = 3k$, show that the frequency equation reduces to $2\alpha^2 - 11\alpha + 11 = 0$ where $\alpha = \dfrac{m\omega^2}{k}$ and $\omega$ is the natural frequency of vibration.

Also determine the natural frequencies of vibration of the system in terms of $k$ & $m$ and the relevant mode shapes.

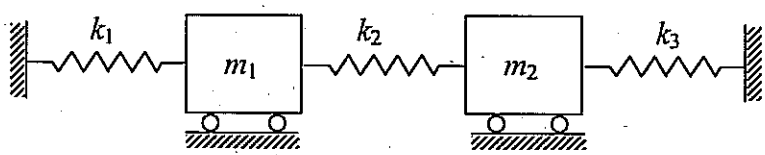



Fig. Q4

**QUESTION 05**

Fig. Q5 shows two torsional systems. System A consists of two rotors each of moment of inertia $I$ and three shaft sections each having a torsional stiffnesses $K$ while the system B consists of a single rotor of moment of inertia $J$ and two shaft sections of torsional stiffnesses $2K$ and $4K$.

Show that, if $J = 6I$ the lower natural frequency of torsional oscillations of system A is equal to that of system B.

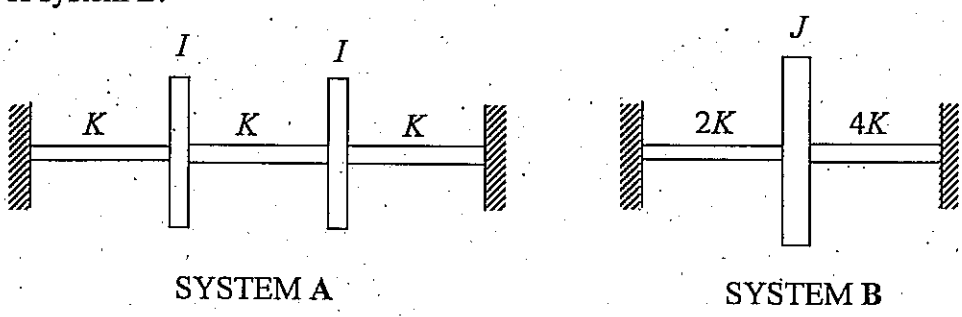



Fig. Q5

**QUESTION 06**

A mechanical system that is at rest is subjected to a unit step input $u(t)$. Its response is given by $y(t) = 1 - e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t$. Determine the following.

(i) Transfer function of the system.

(ii) Input-output differential equation of the system.

(iii) Damped natural frequency, undamped natural frequency and the damping ratio.

(iv) Response of the system to a unit impulse input.

(v) Steady state response for a unit step input.

**QUESTION 07**

(i) Fig. Q7 shows a simplified model of an active engine mount system in which, $m$ is the mass of the engine unit and $k$ & $b$ are the equivalent stiffness and the equivalent viscous damping constant of the active mount respectively.

Derive the equations of motion of the system and obtain the transfer function (with Laplace variable $s$) $\frac{f_o}{f_i}$ for the system.

Show that the magnitude ($M$) of the frequency transfer function is given by $M = \left[\frac{1+4\varsigma^2 r^2}{(1-r^2)^2 + 4\varsigma^2 r^2}\right]^{1/2}$ where $\zeta$ is the damping ratio and $r$ is the ratio of frequencies.

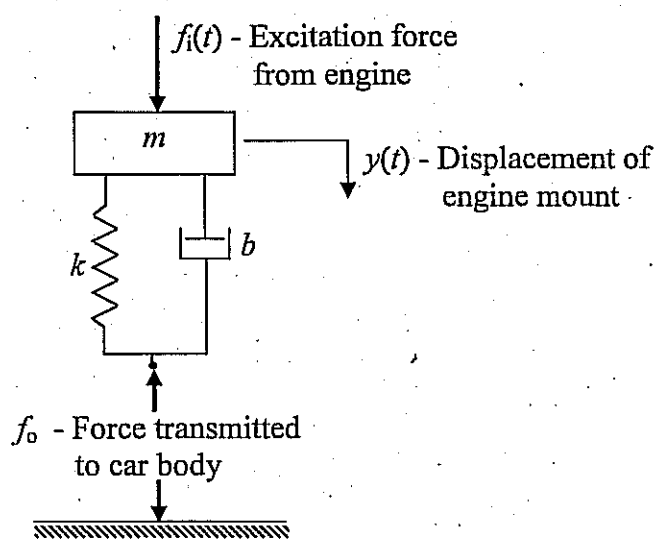



Fig. Q7

(ii) The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{16}{s(s+4)}.$$

Find the unit step response of this system and the peak time & percentage overshoot of the response.

**QUESTION 08**

(i) Draw a typical "Bath tub curve" and explain the variation of failure rate with respect to time in service of a machine.

(ii) What is meant by "Run to failure maintenance"? Under what circumstances would this type of maintenance be appropriate?

# LAPLACE TRANSFORMS

| TIME FUNCTION $f(t)$ | LAPLACE TRANSFORM $F(s)$ |
|---|---|
| Unit Impulse $\delta(t)$ | $1$ |
| Unit step | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $\frac{df(t)}{dt}$ | $sF(s)-f(0)$ |
| $\frac{d^n f(t)}{dt^n}$ | $s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $te^{-at}$ | $\frac{1}{(s+a)^2}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |

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