THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
DIPLOMA IN TECHNOLOGY – LEVEL 4
FINAL EXAMINATION – 2011

MEX4272 – VIBRATION AND FAULT DIAGNOSIS

DATE : 24 th FEBRUARY 2012
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

Fig. Q1 shows a compound pulley which can rotate about its polar axis. A light string connects the spring of stiffness k_1 to the smaller radius r of the compound pulley. One end of another light string is connected to the larger radius $2r$ of the pulley and the other end carries a mass m which is supported on a spring of stiffness k_2 as shown. The polar moment of inertia of the pulley is I.

Determine the equations of motion and derive an expression for the natural frequency for small oscillations of the system.

Evaluate the natural frequency if $k_1 = 1000$ N/m, $k_2 = 500$ N/m, $m = 10$ kg, $r = 50$ mm and $I = 0.12$ kgm^2.

If $m = 10$ kg, $r = 50$ mm and $I = 0.12$ kgm^2 as before, what is the value of k_1 to have a natural frequency of 1.2 Hz with the spring k_2 removed from the system.

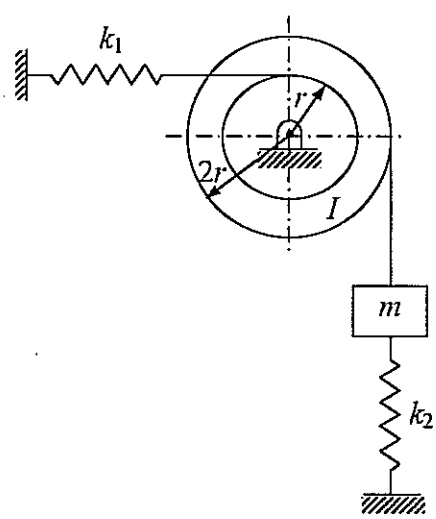


Fig. Q1

QUESTION 02

Fig. Q2 shows a spring-mass-damper system in which the movement of the mass m is restrained by two springs each of stiffness k and a damper of damping coefficient b. Determine the equation of motion.

If m = 10 kg, k = 720 N/m, and b = 120 Ns/m, determine the natural frequency and damping ratio of the system. Determine the free response of the system assuming that the initial conditions are : $x(0) = 0$ and $\dot{x}(0) = 20$ mm/s.

If the damper is now replaced by a new damper having a damping coefficient of 250 Ns/m, what would be the free response of the system? Assume that the initial conditions are, $x(0) = 2$ mm and $\dot{x}(0) = 3$ mm/s.

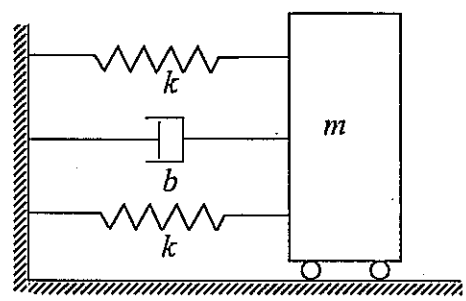


Fig. Q2

QUESTION 03

AB and CD are two uniform rods each of mass m and length L. AB is pivoted at A while CD is pivoted at E. These rods are supported by three springs each having a stiffness k as shown in Fig. Q3. The system is free to oscillate in the vertical plane and the rods are horizontal in the equilibrium position. Determine the equation of motion and the frequency equation for small angular oscillations of the rods.

Also determine the natural frequencies in terms of k and m and the mode shapes.

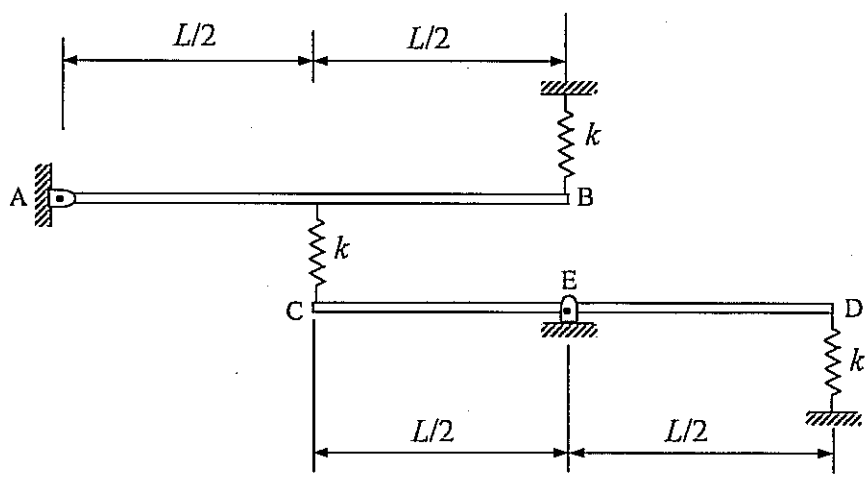


Fig. Q3

QUESTION 04

Fig. Q4 shows a spring-mass system consisting of two masses m & $2m$ and three springs each of stiffness k. Determine the equations of motion of the system and show that the frequency equation can be expressed as $\omega^4 - 3\left(\frac{k}{m}\right)\omega^2 + \frac{3}{2}\left(\frac{k}{m}\right)^2 = 0$, where ω is the natural frequency of vibration.

Also determine the natural frequencies of vibration of the system in terms of k & m and the relevant mode shapes.

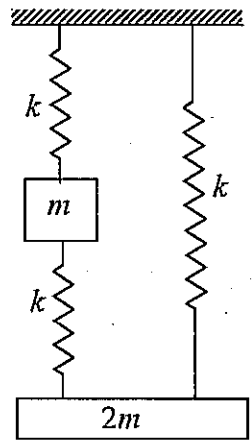


Fig. Q4

QUESTION 05

Fig. Q5(a) shows a torsional system consisting of two rotors of moments of inertia I and $4I$ and two shaft sections having torsional stiffnesses K and $2K$. Show that the frequency equation can be expressed as $\lambda^2 - 1.75\lambda + 0.5 = 0$, where $\lambda = \frac{I\omega^2}{K}$ and ω is the natural frequency of torsional vibration.

If $I = 2$ kgm^2 and $K = 4$ Nm/rad, evaluate the frequencies and determine the mode shapes.

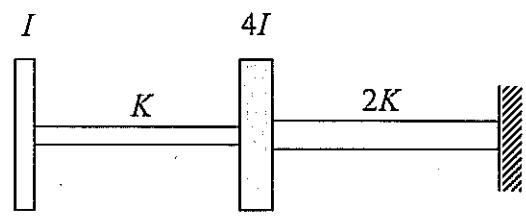


Fig. Q5(a)

Contd..

If the rotor of moment of inertia $4I$ is removed from the system, the system would appear as shown in Fig. Q5(b). Determine the natural frequency of torsional vibration of this modified system.

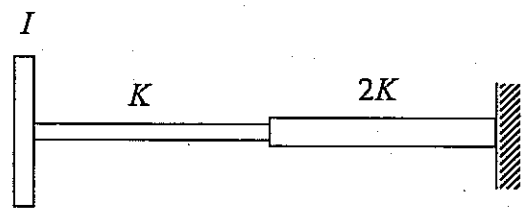


Fig. Q5(b)

QUESTION 06

(a) Fig. Q6(a) shows a unity feedback control system. The unit step response of the system which was initially at rest is $c(t) = 1 - e^{-3t}(\cos 2t + 1.5 \sin 2t)$. Determine the closed loop transfer function of the system and the transfer function $G(s)$.

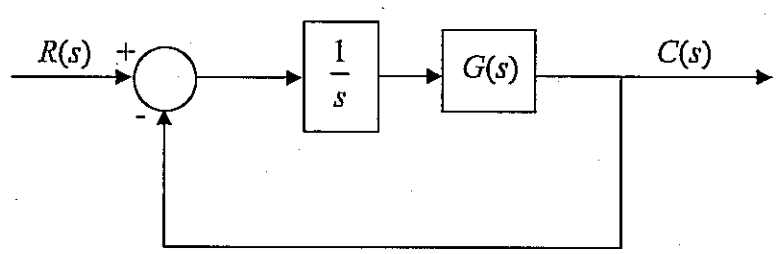


Fig. Q6(a)

(b) Find the Fourier transform of the rectangular pulse signal $x(t)$ defined by

$$x(t) = \begin{cases} 1 & -a < t < a \\ 0 & t < -a, t > a \end{cases}$$ and shown in Fig. Q6(b). Sketch the Fourier transform.

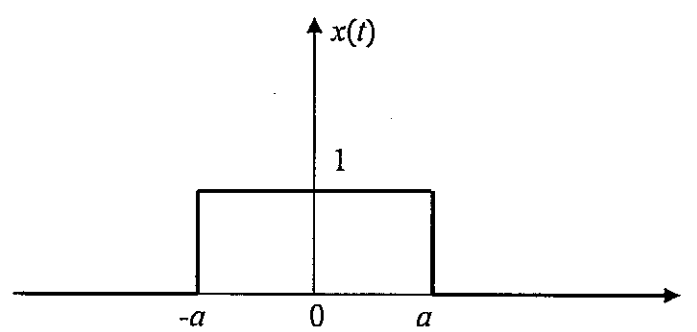


Fig. Q6(b)

QUESTION 07

Fig. Q7 shows a vibration pickup consisting of a mass m, a spring of stiffness k, and a damper of damping coefficient c. All these are housed inside a cage, which is fastened to the vibrating body. With this arrangement, the bottom ends of the spring and the dashpot will have the same motion as the cage (which is to be measured, y) and their vibration excites the suspended mass into motion. The displacement of the mass relative to the cage, $z = x - y$, where x denotes the vertical displacement of the suspended mass, can be measured by attaching a pointer to the mass and a scale to the cage as shown in the figure. The vibrating body is assumed to have a harmonic motion, $y = Y \sin \omega t$

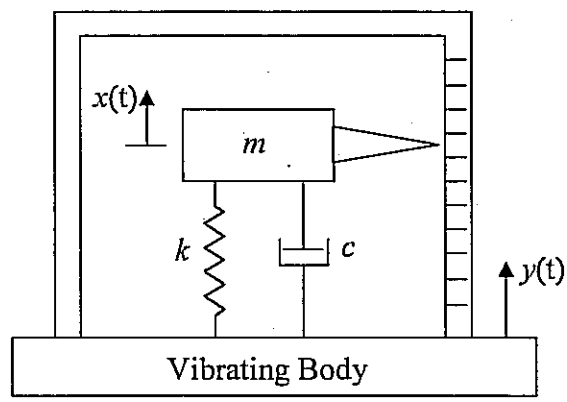


Fig. Q7

Determine the equation of motion of the system and show that the ratio $\left(\frac{Z}{Y}\right)$ is given by

$\frac{Z}{Y} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$, where; $r = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$, $\zeta = \frac{c}{2m\omega_n}$ and Z is the amplitude of z.

QUESTION 08

(i) Explain why mechanical vibration is an important area of study for engineers.

Mechanical vibrations are known to have harmful effects as well as useful ones. Briefly describe three practical examples of good vibration and three examples of bad vibration.

(ii) What is meant by "Run to failure maintenance"? Under what circumstances would this type of maintenance be appropriate?

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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