THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2011/2012

MPZ 3231 – ENGINEERING MATHEMATICS IA

DURATION: THREE (03) HOURS

Date: 04th March 2012	Time: 0930hrs – 1230hrs

Instructions:

- **Answer five (05) questions only, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 04.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

Section A

1. **1.1** Consider the function $f(x) = \frac{x^3}{x^2-1}$, $x \neq \pm 1$.

   1.1.1. Write down the all vertical asymptotes of the graph of $y = f(x)$.

   1.1.2. Find the horizontal and slant asymptotes of the graph of $y = f(x)$.

   1.1.3. Find the sign of $\frac{dy}{dx}$ as x varies and draw the graph of $y = f(x)$.

   1.2 If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

2. **2.1** If $P = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ -1 & 2 & -2 \\ 2 & 2 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

   2.1.1 Prove that $P^T P = I$ and $D = P^T A P$.

   2.1.2 Hence find A^n where $n \in Z^+$.

   2.2 Applying elementary row (column) operations find the factors of $\begin{vmatrix} 1 & a^2 & a^4 \\ 1 & b^2 & b^4 \\ 1 & c^2 & c^4 \end{vmatrix}$.

3 State De Moivre's theorem.

3.1 Using De Moivre's theorem prove that $tan5\theta = \frac{T(T^4-10T^2+5)}{5T^4-10T^2+1}$ where $T = tan\theta$.

3.1.1 Find the general solution of $tan5\theta = 0$.

3.1.2 Using the general solution of $tan5\theta = 0$, write down the non- zero values of $tan^2\theta$ such that $tan5\theta = 0$. Hence obtain the solutions of the equation $T^2 - 10T + 5 = 0$. Hence deduce that $sec^4\frac{\pi}{5} + sec^4\frac{2\pi}{5} = 112$.

3.2 Draw the graphs of the following

3.2.1 $arg\left(\frac{1-z}{1+i}\right) = \frac{\pi}{2}$

3.2.2 $|z-2| = |z-3i|$

4 **4.1** The points P and Q lie on the parabola $y^2 = 4ax$. t_1 and t_2 are the parameters of P and Q respectively. If the line PQ passes through the focus S of the above parabola prove that $t_1t_2 = -1$ and the path of the midpoint of PQ is $y^2 = 2a(x-a)$.

4.2 Find the coordinates of the intersection point of the plane $2x + 4y - 3z - 11 = 0$ and line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$.

4.3 $ABCD$ is a tetrahedron such that each face is an equilateral triangle. If M is the mid point of CD show that the angle between AM and BC is $cos^{-1}\left(\frac{\sqrt{3}}{6}\right)$.

Section B

5 Solve the following differential equations.

5.1 Using an integrating factor, $\frac{dy}{dx} + ycotx = x^{2011}cosecx$.

5.2 Using a suitable substitution, $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$.

5.3 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$ where $y(0) = \left(\frac{dy}{dx}\right)_{x=0} = 4$.

6 Define a unit vector.

Prove that the position vector of the point P which divides $m:n$ the line joining the two points A and B is $\frac{ma+nb}{m+n}$. Where the position vectors of the points A and B are $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively.

6.1 OAB is a triangle such that $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. E lies on the line AB such that $AE:EB = 2:3$ and D is the mid point of OB. If OE and AD intersect at F prove that $\overrightarrow{OF} = \frac{1}{7}(3\boldsymbol{a} + 2\boldsymbol{b})$.

6.2 Explain the geometrical representation of the triple scalar product of vectors.

6.2.1 Prove that the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are coplanar if and only if $\boldsymbol{a} \times \boldsymbol{b} \cdot \boldsymbol{c} = 0$

6.2.2 The position vectors of the points A, B, C, D are $4\boldsymbol{i} + 8\boldsymbol{j} + 12\boldsymbol{k}$, $2\boldsymbol{i} + 4\boldsymbol{j} + 6\boldsymbol{k}$, $6\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$ and $5\boldsymbol{i} + 8\boldsymbol{j} + 5\boldsymbol{k}$. Show that the above points are coplanar.

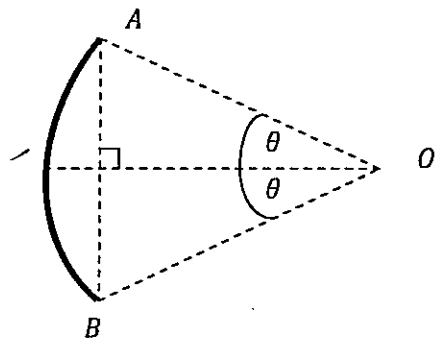
7 **7.1** A student attempts a multiple choice question to which n possible answers are given with one and only one of them being correct. For any question there is a probability of $\frac{\lambda}{\mu}$ that he knows the correct answer. If he does not know the correct answer, then he will mark the answer at random. He does, in fact, mark the correct answer. Prove that the probability that he knew the correct answer is $\frac{\lambda n}{\mu+(n-1)\lambda}$.

7.2 The random variable X has a probability density function given by

$$P(x) = \begin{cases} kx(1-x^2) & , \ 0 \le x \le 1 \\ 0 & , \ Elsewhere \end{cases}$$

k being a constant. Find the value of k and calculate the mean, variance and median of the distribution.

8 8.1



The diagram shows a curved rod AB of length 100 cm which forms an arc of a circle. The end points A and B of the rod are 99 cm apart. The arc AB subtends an angle of 2θ in radians at the center of the circle O.

8.1.1 Prove that $sin\theta = \frac{99}{100}\theta$.

8.1.2 Using a graph and a calculation prove that only one root α lies between $0 \cdot 2$ and $0 \cdot 3$.

8.1.3 Show that $\theta_{n+1} = 50sin\theta_n - 48 \cdot 5\theta_n$ is an iteration formula for $sin\theta = \frac{99}{100}\theta$ and it converges to α in the above interval.

8.1.4 Using the above iteration formula and with the initial value $2 \cdot 5$ find the root α correct to 9 decimal places.

8.2 Using the trapezoidal rule evaluate the following integral with 6 equal intervals. Calculate the absolute error percentage.

$$\int_0^{\frac{\pi}{2}} sin2xdx$$

End

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