00100

THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2011/2012
MPZ 3230 – ENGINEERING MATHEMATICS I
DURATION: THREE (03) HOURS

Date: 25th February 2012	Time: 1330hrs – 1630hrs

Instructions:

- **Answer six (06) questions only, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 05.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ ……

Sign alternate $(+ - + - + \cdots\cdots\cdots)$

- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax -$ ……

Sign alternate in pairs $(+ + - - + + - - + + - - \cdots\cdots\cdots)$

- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax -$ ……

Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots\cdots\cdots)$

- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

SECTION - A

1. Solve the following differential equations.

   1.1. $\frac{dy}{dx} + ytanx = x^{2011}cosx.$

   1.2. $\frac{dy}{dx} = \frac{4x+3y}{3x+4y}x.$

   1.3. $(x+1)\frac{dy}{dx} = (x^2 + 2x + 10).$

2. **2.1** Using a trial function find a particular integral of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 13e^{3x}$. Hence find the general solution of the above differential equation.

   2.2. Using the formula $\frac{1}{D+a}f(x) = e^{-ax}\frac{1}{D}e^{ax}f(x)$ find a particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5sin3x$. Hence find the general solution of the given differential equation.

3. **3.1** Find the Laplace transformation of the function $f(t) = \begin{cases} 3 & , \ 0 < t \leq 5 \\ t^2 & , \ 5 \leq t \end{cases}$.

   3.2. Using the Laplace transformation find the solution of the following differential equation $\frac{d^2y}{dx^2} + y = 3sin2x$, where $x \geq 0$, $y(0) = 1$ and $\left(\frac{dy}{dx}\right)_{x=0} = 2.$

4. **4.1** Find the factors of the following by using row and column operations.

   4.1.1. $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

   4.1.2. $\begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix}$

   4.2. Show that the matrix $\begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ is an orthogonal matrix.

5. Using the augmented matrix and elementary row operations discuss the solutions of the following systems of linear equations.

5.1. $x + 2y - 3z = 4$

$3x - y + 5z = 2$

$4x + y + (\mu^2 - 14)z = \mu + 2$ Where $\mu \in R$

5.2. $x + y = 2$

$x - y = 0$

$3x + y = t$, where $t \in R$.

6. **6.1** Define the Taylor polynomial of degree n.

If $f(x) = \ln(cosx)$ where $\frac{\pi}{2} < x < \frac{\pi}{2}$ prove that $\frac{d^3f(x)}{dx^3} + 2\frac{d^2f(x)}{dx^2}\frac{df(x)}{dx} = 0$.

Hence find the Taylor polynomial of degree four of $f(x) = \ln(cosx)$ about $x = 0$. Deduce that $ln2 \approx \frac{\pi^2}{16}\left(1 + \frac{\pi^2}{96}\right)$.

6.2. Write down the Taylor series expansion of $f(x)$ about $x = a$. Prove that the Taylor series expansion of $f(x) = \frac{1}{x}$ about $x = 3$ is $\sum_{r=0}^{\infty} \frac{(-1)^r(x-3)^r}{3^{r+1}}$.

SECTION - B

7. Define the vector product of two vectors.

7.1. Vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a} \times \mathbf{b} = 3\mathbf{a} \times \mathbf{c}$ .Prove that $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = 3\mathbf{c} + k\mathbf{a}$ where $k \in R$. If $|\mathbf{a}| = 5$, $|\mathbf{b}| = 3$, $|\mathbf{c}| = \sqrt{3}$ and $\mathbf{b} \cdot \mathbf{c} = 0$, then find the values of k and for the above values of k find the angle between

7.1.1. $\mathbf{a}$ and $\mathbf{b}$

7.1.2. $\mathbf{a}$ and $\mathbf{c}$.

7.2. $\vec{OA} = 2\mu\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\vec{OB} = \mu\mathbf{i} + (\mu - 1)\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ Find the area of the triangle OAB and volume of the parallelepiped with the edges OA, OB, OC in terms of μ. Deduce the values of μ such that O, A, B, C are coplanar.

8. **8.1** If $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are the position vector, velocity and acceleration of the particle at time t respectively.

Prove that $\frac{d}{dt}(\boldsymbol{v}\times(\boldsymbol{r}\times\boldsymbol{v})) = 2(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{r} - (\boldsymbol{r}\cdot\boldsymbol{a})\boldsymbol{v} - (\boldsymbol{v}\cdot\boldsymbol{r})\boldsymbol{a}$

8.2. Let $f(t) = \frac{e^t sint}{\sqrt{1+e^{2t}}}\mathbf{i} + \frac{e^t cost}{\sqrt{1+e^{2t}}}\mathbf{j} + \frac{1}{\sqrt{1+e^{2t}}}\mathbf{k}$. Prove that $|f(t)| = 1$ Deduce that

8.2.1. $f(t)$ is perpendicular to $\frac{d}{dt}(f(t))$

8.2.2. $f(t)\cdot\frac{d^2(f(t))}{dt^2} = -\left|\frac{d(f(t))}{dt}\right|^2$.

9. **9.1** Show that the real root of the equation $x^3 - 2 = 0$ lies between 1 and 2.

9.1.1. Prove that $x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$ is an iterative formula to evaluate the numerical value for $\sqrt[3]{2}$ (cubic root of 2). Deduce that x_n converges to $\sqrt[3]{2}$

9.1.2. Taking $x_1 = 1$ and using the above iterative formula, find the sixth iteration of $\sqrt[3]{2}$. Hence obtain the value of $\sqrt[3]{2}$ to two decimal places.

9.2. Using the Jacobi's iteration method, find the sixth iteration of the solution of the following system of linear equations.

$$3x + 10y - z = -8$$
$$2x - 3y + 10z = 15$$
$$10x + y - 2z = 7$$

10. **10.1** The events A and B are such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$ and $P(A\backslash B) = \frac{1}{5}$. Find $P(A \cap B)$, $P(A \cup B)$, $P(A\backslash B)$ and $P(A\backslash B)$. State whether A and B are mutually exclusive or independent. Justify your answer.

10.2. A student attempts a multiple choice question to which 4 possible answers are given with only one of them being correct. For any question there is a probability of $\frac{2}{3}$ that he knows the correct answer. If he does not know the correct answer, then he will mark the answer at random. He does, in fact, mark the correct answer to a question. What is the probability that he knew the correct answer?

11.11.1 The probability density function of a discreet random variable X is

$P(X = x) = k\left(\frac{5}{7}\right)^x$ Where $x = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$. Find the value of constant k, $P(X > 8)$ and $P(5 \leq X < 20)$

11.2. The probability density function of a discreet random variable is given by

x	1	2	3	4	5
$P(X = x)$	$0 \cdot 1$	3λ	2λ	$0 \cdot 3$	$0 \cdot 1$

Find (a) The value of λ (b) $E(X)$ (c) $V(X)$ (d) $E(4X^2 + 3X - 1)$

12. The time taken to fix a certain electronic item, in t hours, has the probability density function $f(t) = \begin{cases} 10kt^2 & , \ 0 \leq t < 0.6 \\ 9k(1-t) & , \ 0.6 \leq t \leq 1.0 \\ 0 & , \ otherwise \end{cases}$, where k is a constant.

12.1. Find the value of k

12.2. Sketch the graph of $f(t)$.

12.3. Write down the most likely time (mode).

12.4. Find the expected time.

12.5. Determine the probability that the time will be

(a) more than 48 minutes, (b) between 24 and 48 minutes.

End

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