00377

THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2011/2012
MPZ 3132 – ENGINEERING MATHEMATICS IB
DURATION: THREE (03) HOURS

Date: 25th February 2012	**Time: 1330hrs – 1630hrs**

Instructions:

- **Answer five (05) questions only, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 05.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

Important integrals

- $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
- $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
- $\int f(x)e^{ax}dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} - \ldots\ldots\ldots$

  Sign alternate $(+-+-+-\ldots\ldots)$
- $\int f(x)\cos axdx = \frac{1}{a}f(x)\sin ax + \frac{1}{a^2}f^1(x)\cos ax - \frac{1}{a^3}f^2(x)\sin ax\ldots\ldots\ldots$

  Sign alternate in pairs $(++--++--++--\ldots\ldots$
- $\int f(x)\sin axdx = -\frac{1}{a}f(x)\cos ax + \frac{1}{a^2}f^1(x)\sin ax + \frac{1}{a^3}f^2(x)\cos ax\ldots\ldots\ldots$

  Sign alternate in pairs after the first term $(++--++--++--\ldots\ldots$
- $\int \sec axdx = \frac{1}{a}\ln|\sec ax + \tan ax|$
- $L(e^{at}) = \frac{1}{s-a}$

Please Turn Over

Section - A

1. **1.1** Define the reduced row echelon form of a matrix.

Consider the matrix $A = \begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ 10 & -7 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

1.1.1. Applying elementary row operations find $RREM(A)$.

1.1.2. Considering $RREM(A)$ solve the following system of linear equations.

$$3x - 2y + z = 0$$
$$-11x + 8y + z = 0$$
$$10x - 7y + z = 0$$
$$-x + y + 2z = 0$$

1.2 A particle moves along the curve $x = t^4 + 2$, $y = t^2$, $z = 3t + 8$ where t is the time. Find the components of its velocity and acceleration at the time $t = 1$ in the direction of $2\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}$.

2. **2.1** Write down the expansion of Taylor series of a given function $f(x)$ about $x = a$

2.1.1. Using the Taylor series expansion prove that $\ln(1 - x) + \sum_{r=0}^{\infty} \frac{x^{r+1}}{r+1} = 0$ where $|x| < 1$.

2.1.2. Using the above formula prove that $ln\left(\frac{3+x}{3-x}\right) \sum_{r=0}^{\infty} \frac{x^{2r+1}}{3^{2r+1}(2r+1)}$. Find the radius of convergence. Hence deduce that $\frac{1}{2} ln2 = \sum_{r=0}^{\infty} \frac{1}{(2r+1)} \left(\frac{1}{3}\right)^{2r+1}$

2.2 $f(x) = x(x + 1) - \pi < x < \pi$ for the other values of x $f(x) = f(2k\pi + x)$ where $k \in \mathbb{Z}$

2.2.1 Draw the graph of the above function in the interval $[-3\pi, 5\pi]$.

2.2.2 Find the Fourier series expansion of the above function.

2.2.3 Deduce that $\sum_{n=1}^{\infty} \frac{1}{r^2} = \frac{\pi^2}{6}$

3. **3.1** The differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 2011cosx - 2012sinx$ has a trial function of the form $y_T = Acosx + Bsinx$.

   3.1.1. Find the values of A and B.

   3.1.2. Find the general solution of the above differential equation.

   3.2. Using the Laplace transformation solve the following system of differential equations with the initial conditions $x(0) = -1$ and $y(0) = 0$.

$$\frac{dx}{dt} = 6x - 3y + 8e^t \qquad \frac{dy}{dt} = 2x + y + 4e^t$$

4. Define the modulus of a complex number.

   4.1. Shade the area satisfying the following both inequalities.

   $|z| \leq 6$ and $-6 \leq Re(z) + Im(z) \leq 6$.

   4.2. Prove that $Log(1 + \cos 2\theta + isin2\theta) = \ln|2cos\theta| + i(\theta + 2k\pi)$ where $k \in Z$.

   4.3. Given that the function such that $g(z) = (1 + i)z^2$, $|z| \leq 1$ and $0 < \arg(z) \leq \frac{\pi}{2}$.

   4.3.1. Find the image of the function $g(z)$.

   4.3.2. Shade the domain and image of the function $g(z)$.

   4.4. Calculate the following limits.

   4.4.1. $\lim_{z\to 0} \frac{Re(z)}{Im(z)}$ as z goes to zero with $y = x$.

   4.4.2. $\lim_{z\to 0} \frac{Re(z)}{Im(z)}$ as z goes to zero with $x = 0$.

   4.5. What can say about the limit of $\lim_{z\to 0} \frac{Re(z)}{Im(z)}$?

Section - B

5. A particle is projected vertically upwards with speed u from a point O. The air resistance on a unit mass is $g\left(\frac{v^4}{u^4}\right)$ where v is the velocity of the particle at a distance x from O. Prove that $\frac{d}{dx}(v^2) = -2g\left(\frac{u^4+v^4}{u^4}\right)$. Deduce that the maximum height attained by the particle is $\frac{\pi u^2}{8g}$. Find the speed of the particle when it returns to O.

6. **6.1** Find the moments of inertia of the following bodies about the given axis.

6.1.1. A uniform rod AB of mass m and length $2a$, the axis perpendicular to the rod passing through the midpoint of the rod.

6.1.2. A uniform circular ring mass m and radius a, the axis perpendicular to the plane of the ring passing through the center.

6.1.3. A uniform circular disc, the axis perpendicular to the plane of the disc passing through the center.

6.2. Using a suitable theorem,

6.2.1. Deduce the moments of inertia of the above rod about the axis perpendicular to the rod passing through A.

6.2.2. Deduce the moments of inertia of the disc of radius a mass M about the axis perpendicular to the plane of disc passing through the point of $3a$ distance of the center of the disc.

6.3. A pendulum consists of a uniform rod AB, of mass m and length $2a$ fixed at B to a point on the circumference of a uniform circular disc of mass M and radius a. The disc and the line of the rod lie in the same vertical plane with the center of the disc lying on the rod produced. The pendulum is free to rotate in a vertical plane about a fixed horizontal axis through A perpendicular to the plane of the disc.

6.3.1. Find the moments of inertia of the pendulum about the axis perpendicular to the plane of the disc passing through A.

6.3.2. When the rod is horizontal the disc is projected with an angular velocity ω in the downward vertical direction. Find the angular velocity after the rod turns at angle θ. If the system rotates complete circles prove that $\omega^2 \geq \frac{12g(m+3M)}{a(8m+57M)}$.

6.3.3. Find the period of small oscillations of the pendulum.

7. Define the binomial distribution.

   7.1. Prove the formulae for the expectation and variance of the above distribution.

   7.2. Explain the normal approximation for the above distribution.

   7.3. It is known that in a sack of mixed paddy seeds 35% are red. Use the normal approximation to the binomial distribution to find the probability that in a sample of 400 seeds there are

      7.3.1. less than 120 red seeds.

      7.3.2. Between 120 and 150 (including 120 , 150) red seeds.

      7.3.3. More than 160 red seeds.

8. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance $\frac{2}{3}\left(\frac{h_1^2+h_1h_2+h_2^2}{h_1+h_2}\right)$from the free liquid surface.

   8.1. The length of a side the square base and height of a thin pyramid are $6a$ and $4a$ respectively. The pyramid is completely filled with a homogeneous liquid with weight w. The weight of the square base is W. The pyramid is kept such that a triangular surface touching a horizontal floor.

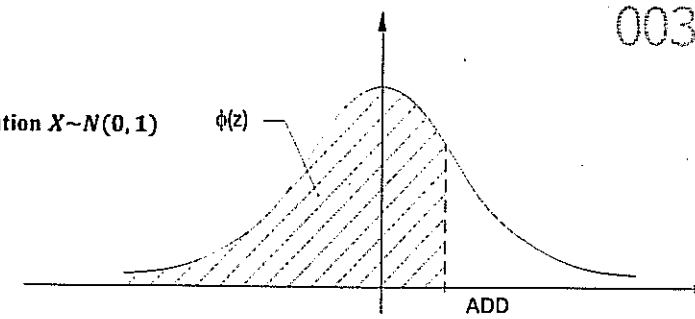
      8.1.1. Find the resultant reaction on the triangular surfaces of the pyramid.

      8.1.2. If the square is a door such that the highest edge of the square is smoothly hinged and the midpoint of the lowest edge is the lock of the door find the reaction on the lock.

End

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The Distribution Function $\emptyset(z)$ of the Normal Distribution $X \sim N(0,1)$



z	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4 8 12	16 20 24	28 32 36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4 8 12	16 20 24	28 32 36
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4 8 12	15 19 23	27 31 35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4 7 11	15 19 22	26 30 34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4 7 11	14 18 22	25 29 32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3 7 10	14 17 20	24 27 31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3 7 10	13 16 19	23 26 29
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	3 6 9	12 15 18	21 24 27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3 5 8	11 14 16	19 22 25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3 5 8	10 13 15	18 20 23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2 5 7	9 12 14	16 19 21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2 4 6	8 10 12	14 16 18
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2 4 6	7 9 11	13 15 17
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2 3 5	6 8 10	11 13 14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1 3 4	6 7 8	10 11 13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1 2 4	5 6 7	8 10 11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1 2 3	4 5 6	7 8 9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1 2 3	4 4 5	6 7 8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1 1 2	3 4 4	5 6 6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1 1 2	2 3 4	4 5 5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0 1 1	2 2 3	3 4 4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0 1 1	2 2 2	3 3 4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0 1 1	1 2 2	2 3 3
2.3	.9893	.9896	.9898	.9901	$.9^2036$	$.9^2061$	$.9^2086$	$.9^2111$	$.9^2134$	$.9^2158$	0 1 1 3 5 8 2 5 7 2 4 6	1 1 2 10 13 15 9 12 14 8 11 13	2 2 2 18 20 23 16 18 21 15 17 19
2.4	$.9^2180$	$.9^2202$	$.9^2224$	$.9^2245$	$.9^2266$	$.9^2286$	$.9^2305$	$.9^2324$	$.9^2343$	$.9^2361$	2 4 6	7 9 11	13 15 17
2.5	$.9^2379$	$.9^2396$	$.9^2413$	$.9^2430$	$.9^2446$	$.9^2461$	$.9^2477$	$.9^2492$	$.9^2506$	$.9^2520$	2 3 5	6 8 9	11 12 14
2.6	$.9^2534$	$.9^2547$	$.9^2560$	$.9^2573$	$.9^2585$	$.9^2598$	$.9^2609$	$.9^2621$	$.9^2632$	$.9^2643$	1 2 3	5 6 7	8 9 10
2.7	$.9^2653$	$.9^2664$	$.9^2674$	$.9^2683$	$.9^2693$	$.9^9702$	$.9^2711$	$.9^2720$	$.9^2728$	$.9^2736$	1 2 3	4 5 6	7 8 9
2.8	$.9^2744$	$.9^2752$	$.9^2760$	$.9^2767$	$.9^2774$	$.9^2781$	$.9^2788$	$.9^2795$	$.9^2801$	$.9^2807$	1 1 2	3 4 4	5 6 6
2.9	$.9^2813$	$.9^2819$	$.9^2825$	$.9^2831$	$.9^9836$	$.9^2841$	$.9^2846$	$.9^2851$	$.9^2856$	$.9^2861$	0 1 1	3 4 4	5 6 6
3.0	$.9^2865$	$.9^2869$	$.9^2874$	$.9^2878$	$.9^2882$	$.9^2886$	$.9^2889$	$.9^2893$	$.9^2896$	$.9^2900$	0 1 1	2 2 2	3 3 4
3.1	$.9^3032$	$.9^3065$	$.9^3096$	$.9^3126$	$.9^3155$	$.9^3184$	$.9^3211$	$.9^3238$	$.9^3264$	$.9^3289$	3 6 9 3 6 8 2 5 7	13 16 19 11 14 17 10 12 15	22 25 28 20 22 25 17 20 22
3.2	$.9^3313$	$.9^3336$	$.9^3359$	$.9^3381$	$.9^3402$	$.9^3423$	$.9^3443$	$.9^3462$	$.9^3481$	$.9^3499$	2 4 7 2 4 6	9 11 13 8 9 11	15 18 20 13 15 17
3.3	$.9^3517$	$.9^3534$	$.9^3550$	$.9^3566$	$.9^3581$	$.9^3596$	$.9^3610$	$.9^3624$	$.9^3638$	$.9^3651$	2 3 5 1 3 4	6 8 10 5 7 8	11 13 14 9 10 12
3.4	$.9^3663$	$.9^3675$	$.9^3687$	$.9^3698$	$.9^3709$	$.9^3720$	$.9^3730$	$.9^3740$	$.9^3749$	$.9^3758$	1 2 3	4 5 6	7 8 9
3.5	$.9^3767$	$.9^3776$	$.9^3784$	$.9^3792$	$.9^3800$	$.9^3807$	$.9^3815$	$.9^3822$	$.9^3828$	$.9^3835$	1 1 2	3 4 4	5 6 7
3.6	$.9^3841$	$.9^3847$	$.9^3853$	$.9^3858$	$.9^3864$	$.9^3869$	$.9^3874$	$.9^3879$	$.9^3883$	$.9^3888$	0 1 1	2 2 3	3 4 5
3.7	$.9^3892$	$.9^3896$	$.9^3900$	$.9^404$	$.9^408$	$.9^412$	$.9^415$	$.9^418$	$.9^422$	$.9^4250$			
3.8	$.9^428$	$.9^431$	$.9^433$	$.9^436$	$.9^438$	$.9^441$	$.9^443$	$.9^446$	$.9^448$	$.9^4500$			
3.9	$.9^452$	$.9^454$	$.9^456$	$.9^458$	$.9^459$	$.9^461$	$.9^463$	$.9^464$	$.9^466$	$.9^4670$			