THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 04

FINAL EXAMINATION – 2012/2013

MPZ4230 – ENGINEERING MATHEMATICS II

DURATION: THREE (03) HOURS

Date: 06th August 2013 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer only six (6) questions.
- State any assumptions you required.
- This paper contains six (6) pages.
- Show all your workings.
- All symbols are in standard notation.

1. Given that the function $f(x,y) = xy^3 + 4x^3y - 16xy + 128$.

   a) Write down all the first and second order partial derivatives of the function $f(x,y)$. [15%]

   i. Show that first order partial derivatives of $f(x,y)$ are zero at the origin, and verify that $f(0,0)$ is neither a maximum nor a minimum value of $f(x,y)$. [15%]

   ii. Find all the maximum and minimum points of the function $f(x,y)$ [30%].

   b) Suppose that the function $f(x,y)$ has an error at most 0.1 unit of each measurement x and y. Estimate the maximum error in calculating the function $f(x,y)$, when $x = 1$ and $y = 1$. [20%]

   c) Find the grad of $f(x,y)$ at the point $(1,1)$ in the direction of the unit vector

   $$\underline{e} = \frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}$$ [20%]

2. a) State the Cauchy Riemann equation for the function $f(z) = u + iv$ to be analytic. [15%]

   b). Show that the function $u = xe^x \cos y - ye^x \sin y$ is a harmonic function. [25%]

   c). Derive the analytic function of the form $f(z) = u + iv$, where v is complex conjugate of $u = xe^x \cos y - ye^x \sin y$. [40%]

   d). Express $f(z)$ of part (c) in terms of z. [20%]

3. a) Consider the following simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + \in_i$ for a data set $(X_i, Y_i), i = 1,2, \ldots . n$. Determine the least squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of β_0 and β_1 respectively. [20%]

   b) Infestation of crops by insects has long been of great concern to farmers and agricultural scientists. The data for age of cotton plant (days) and damaged squares are collected and those are denoted by X and Y.

   i. Fill in the missing values in the following tables of simple linear regression output.

   (Show the way of calculation)

Source	Sum of Squares (SS)	degrees of freedom (d.f)	Mean Square (MS)	F - Statistics
Regression	.....	.....	.....	$F^1_{cal\ 10,0.05} \sim 91.9$
Residuals	.....	.....	.....	
Total	.....	.....		

Variable	Coefficient	S.E (standard error)	t- Statistics
Constant	.....	7.524	.....
X	3.2847	0.3440	.....
n =.....	σ^2 =.....	R^2 =.....	

$\overline{x} = 20.5,\ \overline{y} = 47.67,\ \sum(X_i - \overline{X})^2 = 699,\ F^1_{10,0.05} = 6.9367$ [40%]

   ii. Write down the hypothesis that you would test using the ANOVA table and test the hypothesis at 5% significance level. [10%]

   iii. Test the significance of the coefficients. Use $\alpha = 0.05$ [10%]

   iv. Write down the fitted regression line. Do you think that the fitted model describe the relationship between the two variables? Justify your answer. [20%]

4. a) Studies of the habits of white-tailed deer indicate that these deer live and feed within very limited ranges, approximately 150 to 205 acres. To determine whether the range of deer located in two different geographical areas differ, researches caught, tagged, and fitted 40 deer with small radio transmitters. Several months later, the deer were tracked and identified, and the distance y from the release point was recorded. The mean and standard deviation of the distance from the release point were as given in the following table.

	Location 1	Location 2
Sample size	40	40
Sample mean	2980	3205
Sample standard deviation	1140	963
Population mean	μ_1	μ_2

i. If you have no preconceived reason for believing that one population mean is larger than the other, what would you choose for your alternative hypothesis? and your null hypothesis? [10%]

ii. Would your alternative hypothesis in part (i) imply a one tailed or two tailed test? Explain. [10%]

iii. Do the data provide sufficient evidence to indicate that the mean distance differ for the two geographical locations? test using at 10% significance level. [20%]

iv. Compute the P value if y is standard normal and determine whether to accept or reject H_0. [20%]

b) This experiment based on 7 dogs. Question is whether simulation of the vagus nerve increases levels of immunoreactive insulin in the blood. Two measurements were taken on each dog, one before and other five minutes after simulation. The measurements are blood lead levels of immunoreative insulin.

Dog	Before	After
1	350	420
2	200	130
3	240	250
4	290	310
5	170	280
6	370	450
7	240	280

Define the hypothesis and test it at 5% significance level. (Assume that the mean differences are normally distributed). [40%]

5. a) Using the 4th order Taylor's series method, find $y(1.1)$ and $y(1.2)$ by solving following second order ordinary differential equation: [50%]

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} - x^3 = 0, \qquad y(1) = 1 \text{ and } \left(\frac{dy}{dx}\right)_{x=1} = 1$$

b) i. Solve the following differential equation using second order Runge – Kutta method

$$2\frac{dy}{dx} = (1+x)y^2, \qquad y(0) = 1$$

and hence find $y(0.1)$ and $y(0.2)$ [30%]

ii Find the exact value of $y(0.1)$ and $y(0.2)$. [20%]

6. a) Classify the elliptic, parabolic and hyperbolic form of the second order partial differential equation. [10%]

b) i. Suppose that a point O is at the center and A, B, P and Q are surrounding points such that PO = QO = h, OA = ah and OB = bh, $0 \le a \le 1$, $0 \le b \le 1$. If lines OP and OA are parallel to the X axis and OQ and OB are parallel to the Y axis. Using Taylor Series expansion, show that [35%]

$$\frac{\partial^2 u}{\partial x^2} = \frac{2}{h^2}\left[\frac{1}{a(1+a)}u_A + \frac{1}{1+a}u_P - \frac{1}{a}u_O\right]$$

and

$$\frac{\partial^2 u}{\partial y^2} = \frac{2}{h^2}\left[\frac{1}{b(1+b)}u_B + \frac{1}{1+b}u_Q - \frac{1}{b}u_O\right]$$

ii. Hence derive the five point scheme for the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

[10%]

c) Solve the Laplace equation defined in the region and the boundary values shown in *Figure 1*, and write down the solution in matrix form and hence find the solution at the points $u_{1,1}, u_{1,2}$ and $u_{2,1}$

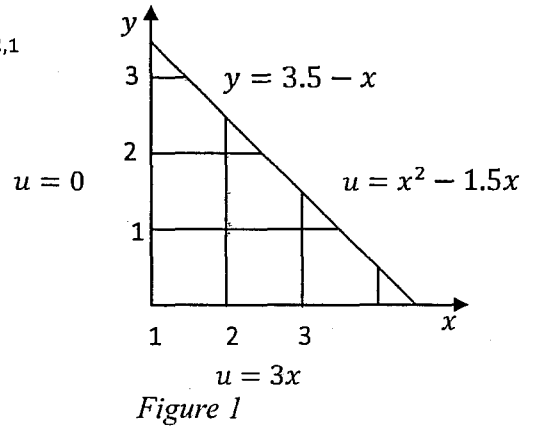


Figure 1

[45%]

7. In the series RLC circuit, inductance $(L) = 20$ Henry, resistance $(R) = 80$ ohms, capacitance $(C) = 10^{-2}$ farad and voltage $(V_0) = 100\cos 2t$ volts.

   a) Show that capacitor charge at any time t is,

$$q = e^{-2t}\left[-\frac{1}{13}\cos t - \frac{18}{13}\sin t\right] + \frac{1}{13}\cos 2t + \frac{8}{13}\sin 2t$$

   [80%]

   b) Using above result find current and capacitor voltage at time t. [20%]

(Assume that,

Kirchhoff's voltage law for series circuit yield

$V_L + V_R + V_C = V_0$, where

$V_L = L\frac{di}{dt}$, $V_R = iR$, $V_C = \frac{1}{C}\int idt$, $i = \frac{dq}{dt}$)

8. a) Given that the vectors $\underline{u}_1 = (1,0,0,0)$, $\underline{u}_2 = (1,2,0,1)$, $\underline{u}_3 = (1,1,1,1)$ and $\underline{u}_4 = (1,0,0,1)$ and $\underline{u}_i \in \mathbb{R}^4$, $i = 1,\ldots,4$.

   i. Show that the vectors $\underline{u}_1, \underline{u}_2, \underline{u}_3$ and $\underline{u}_4$ are linearly independent vectors on $\mathbb{R}^4$ and those vectors span $\mathbb{R}^4$. [20%]

   ii. Consider the $\mathbb{R}^4$ as a real inner product space with Euclidean inner product. Find the orthogonal vectors of the above vectors using Gram Schmidt process. [30%]

   iii. Using part (ii) verify the following identity for any two orthogonal vectors $\underline{u}$ and $\underline{v}$.

$$\|\underline{u} + \underline{v}\|^2 = \|\underline{u}\|^2 + \|\underline{v}\|^2$$

   [10%]

   b) T is a linear transformation of $\mathbb{R}^2$ to $\mathbb{R}^2$, such that

$$T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}7\\8\end{pmatrix}$$

   Determine the following:

   i. Transformation, T. [30%]

   ii. Kernel of T, *Ker T*. [10%]

). Let $A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & -1 \end{pmatrix}$

a) Find the eigen values of A and its corresponding eigen vectors. [35%]

b) Find an orthogonal matrix P such that $D = P^{-1}AP$ where D is a diagonal matrix. [25%]

c) Using part (b) prove that, $D = P^{-1}A^{-1}P$ and $D^n = P^{-1}A^nP$ [15%]

d) Derive the quadratic functions of $Q(\underline{x}) = \underline{x}^T A\underline{x}$ and $Q(\underline{y}) = \underline{y}^T D\underline{y}$.

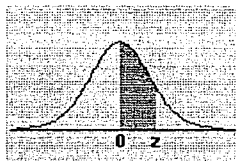
where $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ [10%]

e) Show that $P^{-1} = P^T$, and hence Show that $Q(\underline{x}) = Q(\underline{y})$ where $\underline{y} = P^T\underline{x}$ [15%]

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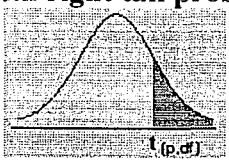
Standard Normal (Z) Table

Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905