The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Diploma in Technology/Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX4272 Vibration and Fault Diagnosis**
Academic Year	:	2012/13
Date	:	09th August 2013
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

QUESTION 01

Fig. Q1 shows a rigid, weightless rod PQ of length $3a$ pivoted at its left end P. The rod is supported by two springs of stiffnesses k_1 and k_2 attached to the rod at points at distances of a and $2a$ respectively from the left end. A spring of stiffness k_3 and carrying a mass m is attached to the right end Q of the rod. The rod is horizontal in the equilibrium position.

(a) Determine the natural frequency of vibration of the system.

(b) If $k_1 = 2k$ and $k_2 = k_3 = k$, show that the natural frequency of vibration of the system is the same as that of a mass m supported on a spring of stiffness $0.4k$.

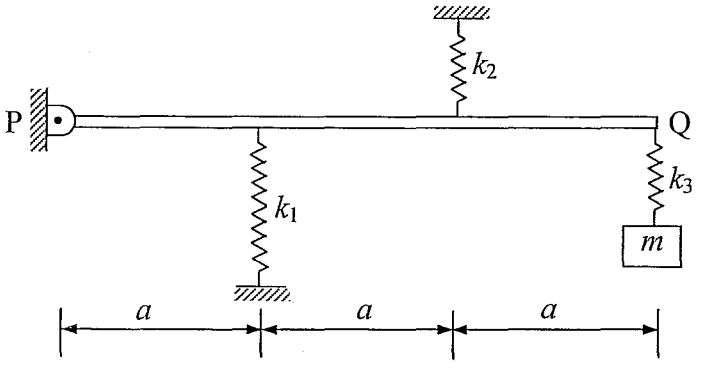


Fig. Q1

QUESTION 02

Fig. Q2 shows a spring-mass-damper system in which the movement of the mass m is restrained by a spring of stiffness k and two dampers of damping coefficients b_1 and b_2.

(a) Determine the equation of motion.

(b) If $m = 2$ kg, $k = 450$ N/m, $b_1 = 10$ Ns/m and $b_2 = 20$ Ns/m, determine,

(i) the natural frequency and damping ratio of the system.

(ii) the free response of the system assuming that the initial conditions are : $x(0) = 0$ and $\dot{x}(0) = 30$ mm/s.

(iii) If b_2 is changed to 50 Ns/m, what would be the free response of the system? Assume that the initial conditions are: $x(0) = 0.2$ mm and $\dot{x}(0) = 1$ mm/s.

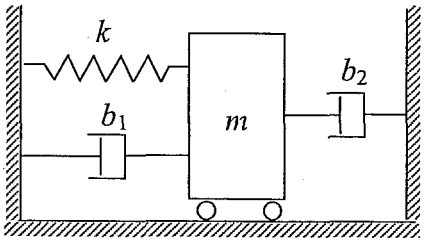


Fig. Q2

QUESTION 03

Fig. Q3 shows a compound pulley which can rotate about its polar axis. One end of a spring of stiffness k is fixed to the smaller radius r of the compound pulley while the other end is attached to a fixed support. One end of another spring of stiffness k is fixed to the larger radius $2r$ of the pulley while the other end carries a mass m which is supported on a spring of stiffness $2k$ as shown. The polar moment of inertia of the pulley is I.

(a) Determine the equations of motion and the frequency equation for small oscillations of the system.

(b) Show that, if $I = mr^2$, the frequency equation reduces to $\alpha^2 - 8\alpha + 11 = 0$, where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of oscillations of the system.

(c) Determine the natural frequencies in terms of k and m.

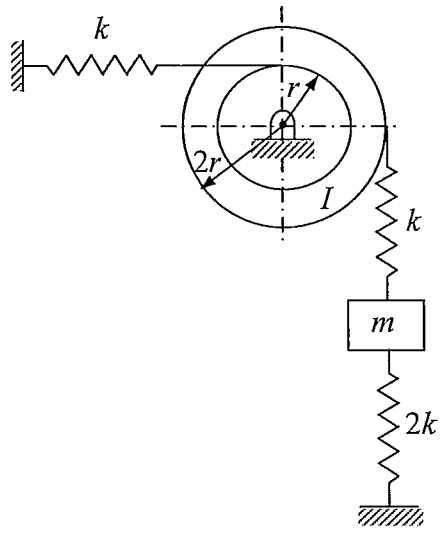


Fig. Q3

QUESTION 04

Fig. Q4 shows a spring-mass system consisting of two masses m_1 and m_2 and four springs of stiffnesses k_1, k_2, k_2 and k_3.

(a) Derive the equations of motion and express them in the form of a matrix.

(b) Obtain the frequency equation (You do not need to simplify this equation).

(c) If $k_1 = k_2 = k$, $k_3 = 2k$, $m_1 = m$ and $m_2 = 3m$,

(i) show that the frequency equation can be expressed as

$$\omega^4 - \frac{10}{3}\left(\frac{k}{m}\right)\omega^2 + \frac{7}{3}\left(\frac{k}{m}\right)^2 = 0,$$

where ω is the natural frequency of vibration.

(ii) determine the natural frequencies of vibration of the system in terms of k and m and the ratio of amplitudes of motion of the two masses corresponding to each frequency.

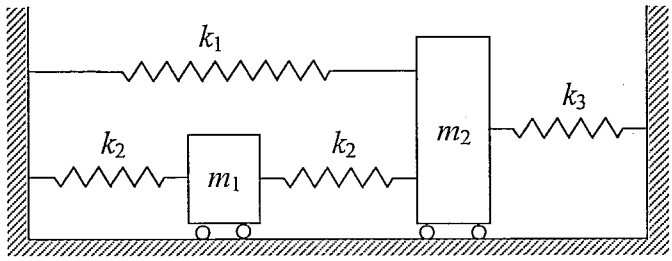


Fig. Q4

QUESTION 05

Fig. Q5 shows a torsional system consisting of three rotors, each of moment of inertia I and three shaft sections having torsional stiffnesses $2K$, K and K.

(a) Derive the equations of motion and obtain the frequency equation.

(b) If, I = 1 kgm^2 and K = 2 Nm/rad, determine the natural frequencies of torsional oscillations and the mode shapes corresponding to the lowest and the highest frequencies of the system.

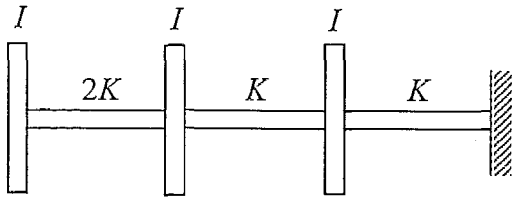


Fig. Q5

QUESTION 06

(a) A mechanical system is given by the equations of motion,

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

Obtain the natural frequencies and the mode shapes of the system.

(b) The open loop transfer function of a unity, negative feedback control system is given by

$$G(s) = \frac{36}{s(s+6)}$$

Find the unit step response of the system. Assume that all initial conditions are zero.

QUESTION 07

Fig. Q7 shows a simplified model of an active engine mount system in which,

f_i – excitation force from the engine,

f_o – force transmitted to the passenger compartment,

y – displacement of the engine mount with respect to a frame fixed to the passenger compartment,

m – mass of the engine unit,

k – equivalent stiffness of the active mount,

b – equivalent viscous damping constant of the active mount.

(a) Derive equations of motion of the system.

(b) Determine the transfer function $\frac{f_o}{f_1}$ for the system and show that it can be expressed as,

$$G(s) = \frac{2\varsigma\omega_n s + \omega_n^2}{(s^2 + 2\varsigma\omega_n s + \omega_n^2)}$$

where ω_n is the undamped natural frequency and ζ is the damping ratio.

(c) Obtain the frequency transfer function and show that the magnitude M of the transfer function, for $\zeta = 0.5$, is given by

$$M = \sqrt{\frac{1+r^2}{1-r^2+r^4}}$$

where $r = \frac{\omega}{\omega_n}$ and ω is the excitation frequency.

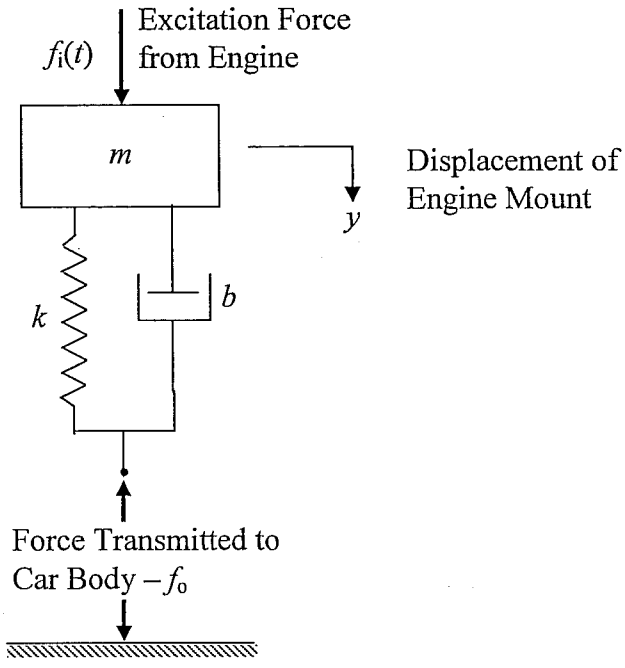


Fig. Q7

QUESTION 08

(a) Draw a typical "Bath tub curve" and explain the variation of failure rate with respect to time in service of a machine.

(b) Write short notes on the following.

(i) Measurement accuracy

(ii) Instrument accuracy

(iii) Precision

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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