THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY – LEVEL 03
FINAL EXAMINATION – 2012/2013
MPZ3231 – ENGINEERING MATHEMATICS IA
DURATION: THREE (03) HOURS

Date: 04th August 2013	Time: 0930hrs – 1230hrs

Instructions:

- **Answer five (05) questions only, selecting at least two(02) questions from each section A & B.**
- **The paper contains 06 pages.**
- **Bold or Underline letters represent the vectors.**
- **All symbols are in standard notation.**

Section A

1. **(a)** Solve the equation $|A - \lambda I| = 0$ for λ values :

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ [15%]

(b) If $\Delta = \begin{vmatrix} 1 & bc & bc\,(b+c) \\ 1 & ca & ca\,(c+a) \\ 1 & ab & ab\,(a+b) \end{vmatrix}$ using row or column operations, show that

$\Delta = abc \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$, and hence evaluate Δ. [30%]

(c) Find the values of p for which the following system of linear equations has non-trivial solution.

$3x + y + z = px \;\; ; \;\; x + 3y + z = py \;\; ; \;\; x + y + 3z = pz$ [20%]

(d) Consider the system of equations:

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

Find values of λ and μ for which the above system of equations has

(i) a unique solution.

(ii) many solutions.

(iii) no solutions. [35%]

2. (a) Consider the function $f(x) = \frac{x^2-3x}{x+1}$, $x \in \mathbb{R} \setminus \{-1\}$.

(i) Write down the horizontal, vertical and slant asymptotes of the graph of $y = f(x)$. [15%]

(ii) Find the sign of $\frac{dy}{dx}$ as x varies and draw the graph of $y = f(x)$. [45%]

(b) If Z is a function of x and y and $x = e^U + e^{-V} \;\; ; \;\; y = e^{-U} + e^V$

Prove that $\frac{\partial Z}{\partial U} - \frac{\partial Z}{\partial V} = x\frac{\partial Z}{\partial x} - y\frac{\partial Z}{\partial y}$ [20%]

(c) Show that the equation:

$$\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} = 0, \text{ is satisfied by, } Z = \ln\sqrt{(x^2 + y^2)} + \frac{1}{2}tan^{-1}\left[\frac{y}{x}\right]. \qquad [20\%]$$

3. **(a)** Show that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P\,(at^2, 2at)$ is $y + tx = 2at + at^3$. The normal at P meets the x axis at G and the midpoint of PG is N. [20%]

(i) Find the equation of the locus of N as P moves on the parabola. [30%]

(b) Prove that the line $px + qy + r = 0$ is a tangent to the ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ if } a^2p^2 + b^2q^2 = r^2. \qquad [15\%]$$

(c) Find the parametric equation of the line passing through the points Q_0 and Q_1 :

where $Q_0 \equiv (-1, 2, 2)$ and $Q_1 \equiv (1, 3, -1)$. [10%]

(d) Find the co-ordinates of the point of intersection of the line Q_0Q_1 with the plane $x + 2y + 4z = 7$. [25%]

4. **(a)** Suppose that an equation of a curve is given by $\left|\frac{Z-2}{Z+2}\right| = 2$,

where Z is a complex number. Show that locus of the curve is a circle and hence determine its center and radius. [25%]

(b) State DeMoivre's theorem. [10%]

(i) If $Z = \cos\theta + i\sin\theta$ then prove that

$$Z^n + \frac{1}{Z^n} = 2\cos n\theta \text{ and } Z^n - \frac{1}{Z^n} = 2i\sin n\theta \qquad [10\%]$$

(ii) Prove the following:

(A) $4\cos^3\theta = \cos 3\theta + 3\cos\theta$ and

(B) $4\sin^3\theta = 3\sin\theta - \sin 3\theta$ [30%]

(c) Let Z and Z_1 be two complex numbers. Such that $Z = ce^{i\theta}$; where c is a constant.

If $\left[\frac{Z+c}{Z-c}\right]^2 = \frac{Z_1 + 2c}{Z_1 - 2c}$ then

prove that $Z_1 = 2c\ \cos\theta$. [25%]

Section B

(a) Show that the general solution of the differential equation:

$x\,[y-x]\frac{dy}{dx} = y\,[y+x]$ is

$\frac{y}{x} - ln\,xy = c$

where c is an arbitrary constant. [35%]

(b) Find the value of λ, for which the following differential equation is exact.

$(x\,y^2 + \lambda x^2 y)\,dx + (x+y)\,x^2\,dy = 0;$

and hence solve the equation for this value of λ. [35%]

(c) Find the general solution of the differential equation:

$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 12\frac{dy}{dx} = 0.$ [30%]

6. **(a)** Define the ***scalar product*** and ***vector product*** of two vectors. [10%]

(b) The position vectors of three points in the plane A, B and C are

$\boldsymbol{a} = 2\lambda\,\underline{i} + 3\,\underline{j} + 2\,\underline{k}$; $\boldsymbol{b} = \lambda\,\underline{i} + (\lambda-1)\,\underline{j} + \underline{k}$ and $\boldsymbol{c} = 4\,\underline{i} + 2\,\underline{j} + 3\,\underline{k}$;

where λ is a scalar parameter.

(i) Explain whether the points A, B and C are collinear. [20%]

(ii) Find the value of λ such that $\boldsymbol{a}$ and $\boldsymbol{c}$ are perpendicular. [20%]

(iii) Determine the values of λ such that the points A, B and C are coplanar. [30%]

(iv) When $\lambda = 2$, find the volume of the parallelepiped formed by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [20%]

7. **(a)** There are two modes of transporting material from Colombo to a city in the south, namely, by land or sea. Land transportation may be by rail or highway. About 70% of the material is transported by land and the rest is transported by sea. Also 60% of all land transportation is by highway and the rest by rail. The percentages of damaged cargo are 10 % by highway, 6 % by rail and 5% by sea.

(i) What percentage of the total cargo may be expected to be damaged? [20%]

(ii) If a damaged cargo is received what is the probability that it was shipped by Land? [20%]

(b) Suppose that Y possesses the density function

$$f(y) = \begin{cases} cy\,; 0 \leq y \leq 2 \\ 0\,; \quad o/w \end{cases}$$

(i) Find the value of c that makes $f(y)$ a probability density function. [10%]

(ii) Find the cumulative distribution function $F(y)$. [10%]

(iii) Graph of $f(y)$ and $F(y)$. [20%]

(iv) Find $E(y)$ and $V(y)$. [20%]

8 **(a)** Using the Bisection method,

(i) Prove that a negative root of equation,

$x^3 - 4x + 9 = 0$ lies between -3 and -2. [10%]

(ii) Let x_n be the n^{th} iterative solution of the above equation for the given initial values $x_0 = -3$ and $x_1 = -2$. Find the value of x_3. [10%]

(b) Using Newton-Raphson method, compute approximately the square root of 24, using 5 iterations. (Take $x_0 = 1$) [20%]

(c) The values of y with respect to x are tabulated as follows:

x	0	2	4	6
y	1	15	85	259

(i) Construct the difference table.

(ii) Find the polynomial of degree three. [30%]

(d) A function $f(x)$ is defined as given in the table below:

x	10	20	30	40
$f(x)$	1.2	2.0	4.5	8.0

Using suitable interpolation technique, compute $f(35)$. [30%]

End

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