THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION - 2012/2013

MPZ 3132 – ENGINEERING MATHEMATICS IB

DURARTION: THREE HOURS (3.00)

Date: 27th July 2013 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer five (05) questions only, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 05.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

Important integrals

$$\int e^{ax}\, sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$$

$$\int e^{ax}\, cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$$

$$\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Sign alternate $(+-+-+-+-+\ldots\ldots\ldots)$

$$\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \ldots\ldots\ldots\ldots\ldots$$

Sign alternate in pairs $(++--++--++--++--++--\ldots\ldots\ldots)$

$$\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \ldots\ldots\ldots\ldots\ldots$$

Sign alternate in pairs after the first term $(++--++--++--\ldots\ldots\ldots)$

Section A

1. **(a)** State the Dirichlet's conditions and write down the Fourier series expansion Of a period function $f(x)$. [25%]

A function $f(x)$ is defined such that $f(x) = \begin{cases} 3 + 2x - x^2 & \text{for } -\pi < x < \pi \\ f(x + 2\pi) & \text{for } x \leq -\pi \text{ or } x \geq \pi \end{cases}$.

Find the Fourier series expansion of $f(x)$. [30%]

Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. [20%]

(b) Write down the Taylor polynomial of $f(x)$ about $x = k$. [05%]

i) The function $f(x)$ is defined such that $f(0) = 1$ and $\frac{df}{dx} = xf(x) + 1$. Find the values of $\left(\frac{d^r f}{dx^r}\right)_{x=0}$ for $r = 1, 2, 3, 4$. [15%]

ii) Hence deduce the order four Taylor polynomial of $f(x)$ about $x = 0$. [05%]

2. **(a)** Prove that

$$\frac{1}{D + \alpha} f(x) = e^{-\alpha x} \frac{1}{D} e^{\alpha x} f(x).$$ [25%]

i) Find a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 7y = e^{4x} \cos(4x).$$ [35%]

ii) Hence, obtain the general solution of the above differential equation. [15%]

(b) Using the elementary operations find the rank of $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 5 & 8 & 8 \end{bmatrix}$. [25%]

3. Define the "Laplace transformation" of a function: [10%]

(a) Find the "Laplace transformation" of the function defined as

$$f(x) = \begin{cases} 0 & \text{if} \quad 0 < x < 2, \\ 3x^2 & \text{if} \quad 2 \leq x. \end{cases}$$ [40%]

(b) Using the "Laplace transformation" solve the following differential equation.

$$\frac{d^2y}{dx^2} + 16y = sin3x \text{ where } y(0) = y'(0) = 0.$$ [50%]

4. **(a)** Shade the regions satisfying the following inequalities

$|z - (1+i)| \leq 1$ and $-1 \leq Im(z) - Re(z) \leq 1$. [20%]

(b) Find $Log(1+i)$ of the form $x + iy$ where $x, y \in \mathbb{R}$ [20%]

(c) The function f is defined as follows

$$f(z) = \frac{z^2}{1+i}, |z| \leq 1 \text{ and } 0 < \arg(z) \leq \frac{\pi}{2}.$$

i) Determine the image of the above function. [20%]

ii) Draw the domain and image of the above function. [20%]

(d) The function g is defined as

$g(z) = \frac{z+1}{2z+1}$. Show that $1-1$ and find the inverse of g. [20%]

Section B

5. A particle is projected upwards with speed u from a point O along a greatest slope line of an inclined plane at an angle $sin^{-1}\left(\frac{1}{n^2}\right)$ (where $n > 1$) to the horizon. The resistance to the motion on a unit mass is $g\left(\frac{v^4}{u^4}\right)$ where v is the velocity of the particle at a distance x from O.

(a) Prove that

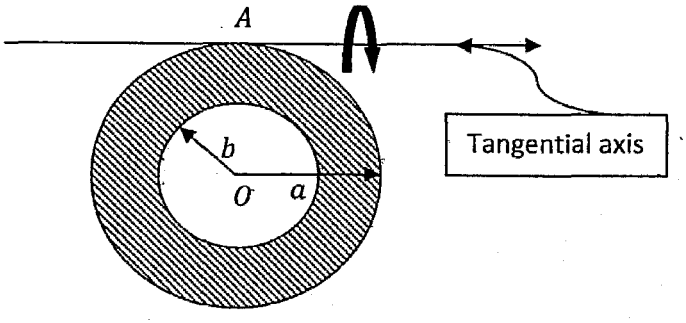
$$\frac{d}{dx}(v^2) = -2g\left(\frac{u^4 + n^2v^4}{n^2u^4}\right).$$ [20%]

(b) Hence deduce the maximum distance traveled by the particle in the upward direction. [30%]

(c) Find the speed of the particle when it returns to O with the above resistance. [50%]

6. **(a)** Find the moments of inertia of a uniform circular ring of radius a and mass m about the perpendicular axis to the plane of the ring passing through the center. [10%]

(b) A uniform lamina in the form of circular annulus center O, mass M, outer radius a and inner radius b is shown below. Find the moments of inertia of the lamina about an axis through O perpendicular to the plane of the lamina.



i) Find the moments of inertia of the lamina about its diameter. [30%]

ii) Find the moments of inertia of the lamina about the tangential axis. (refer the figure). [10%]

iii) Now the circular annulus is free to rotate about the tangential axis. When the circular annulus is vertical it is projected with an angular velocity ω in the downward vertical direction.

A. Find the angular velocity after the circular annulus turns at angle θ. [25%]

B. Find the period for the small oscillation of the circular annulus. [25%]

7. **(a)** With the usual notation, show that the moment generating function of the normal distribution is $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$. [25%]

(b) Using part **(a)**, find the mean and variance of the normal distribution. [20%]

(c) The masses of mangos are normally distributed. 5% of the mangos have a mass greater than $85g$ and 10% have a mass less than $25g$.

i) Find the mean and the variance of the distribution. [40%]

ii) Calculate the symmetrical limits about the mean, within which 75% of the masses lie. [15%]

8. **(a)** A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances from the surface of the liquid to AB and CD are h and H respectively. Prove that that the center of pressure of the lamina is at the distance of $\frac{2}{3}\left(\frac{h^2+hH+H^2}{h+H}\right)$ from the free liquid surface. [30%]

The length of the square base and height of a thin pyramid are $6a$ and $4a$ respectively. The pyramid is completely filled with a homogeneous liquid with weight w. The weight of the square base is W. The pyramid is kept such that a triangular surface touching a horizontal floor.

i) Find the resultant reaction on the triangular surfaces of the pyramid. [50%]

ii) Suppose that the square base is a door such that the highest edge of the square is smoothly hinged and the midpoint of the lowest edge is the lock of the door. Find the reaction on the lock. [20%]

END

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