THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY – FOUNDATION (LEVEL 01)
FINAL EXAMINATION - 2007
MPZ 1331 – APPLIED MATHEMATICS II
DURATION – THREE (03) HOURS

DATE : 27th April 2008 **TIME: 9.30-12.30 hrs.**

ANSWER SIX (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS. YOU CAN'T USE MOBILE PHONES AS CALCULATORS.

SECTION – A

01. Due to track repairs a train retards uniformly, with retardation $1ms^{-2}$ from a speed of $40ms^{-1}$ at A to the speed of $10\ ms^{-1}$ at B. The train travels from B to C, a distance of 3.5 km, at a speed of $10ms^{-1}$ and then accelerates uniformly, with acceleration $0.2ms^{-2}$ so that its speed at D is $40\ ms^{-1}$.

Sketch the velocity-time graph for the journey from A to D. Show that the distance from A to D is 8km. Also show that the journey from A to D takes 330 seconds more than it would if the train traveled at a constant speed of $40\ ms^{-1}$ from A to D.

02. A ship 'A' whose full speed is 40 km/h is located at 20km due west of a ship 'B' which is traveling uniformly with speed 30km/h in the direction due north. The ship A travels at full speed in a direction chosen so as to intercept B as soon as possible. Find this direction of 'A' and calculate to the nearest minute the time A would take to reach B. If after the half of this time has elapsed due to an engine failure of ship A it proceeds thereafter at half of its maximum speed. Find the course (direction) which A should then set in order to approach as close as possible to B , and calculate the distance of closest approach (in kilometers to 2 decimal points).

03. (a) At t=0 a particle is fired from the origin O with speed V at an angle of inclination α above the x axis, which is horizontal, At the instant t = t when the particle is at the highest point of its path, a second particle is fired from O with speed U at angle of inclination β above the horizontal x axis so that the particles collide at a point P on the x axis at time t = T.

Show that $t_1 = \frac{V \sin\alpha}{g}$, $T = \frac{2vSin\alpha}{g} = \frac{v}{g}Sin\alpha + \frac{2u}{g}Sin\beta$

$OP = 2u^2/g\, Sin\alpha Cos\alpha = 2V^2/g\, Sin\beta Cos\beta$

Hence show that $\text{Tan}\ \beta = \frac{1}{4}Tan\alpha$,

Show also that if Tan α =3, then $8u^2 = 5V^2$

(b) Particle of mass M rests on a rough horizontal plane, the coefficient of friction being $\frac{1}{3}$. The particle is connected by a light inextensible string passing over a smooth pulley at the edge of the plane to a particle of mass m which hangs vertically. Find the condition that the system should remain at rest when released.

04. (a) Three identical spheres A,B,C lie at rest in a straight line on a smooth horizontal table. Particle A is projected towards B with speed U. After the collision between A and B, B moves onto hit C. The coefficient of restitution between any two of the spheres is ½. Find the speeds of the spheres A and B after first collision. Hence deduce the speeds of the spheres after the second collision and state whether their will be a third collision.

(b) The frictional resistance to the motion of a car of mass 1000 kg is 30υN, where υms^{-1} is the speed of the car. The car ascends a hill of inclination $\text{Sin}^{-1}(1/10)$, the power exerted by the engine being 12.8kw. If the car is moving at a steady speed, prove (do not merely verify) that this steady speed is $10ms^{-1}$, on reaching the top of the hill the road becomes flat, find the immediate acceleration of the car.
($g=9.8ms^{-2}$)

SECTION – B

05. The centre of a regular hexagon ABCDEF of side 'a' is O. Forces of magnitudes P,2P,3P,4P,nP and mP act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{FA}$ respectively. The resultant of these six forces is of magnitude 3P, acting in a direction parallel to EF.

i. determine the values of m and n.

ii. show that the sum of the moments of the forces about O is $9Pa\sqrt{3}$ of the mid point of EF is M.

iii. Find the equation of the line of action of the resultant, with reference to OM as x axis and OA as y axis.

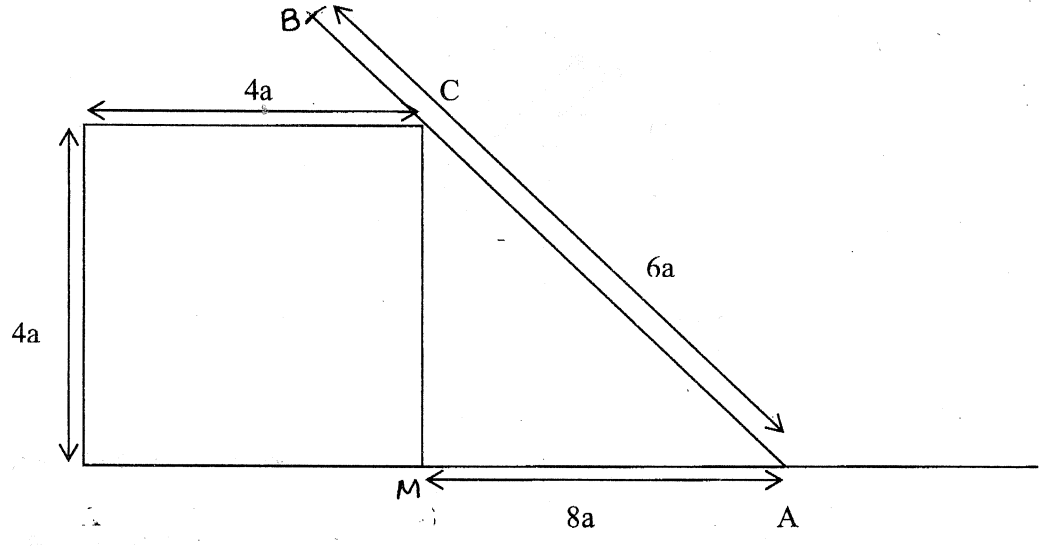
06. A uniform cube of edge 4a and weight W rests on a smooth horizontal surface (see diagram). A uniform smooth rod AB, length 6a and weight w. has one end freely hinged to a point A of the surface. The rod rests against the mid point of a horizontal edge of the top face of the cube. The road is in a vertical plane through the centre of the cube. A light inelastic string of length 3a joins A to the mid point M of the horizontal edge of the cube nearest to A.

(i) By considering the equilibrium of the rod, show that the magnitude of the reaction between the rod and the cube is $\frac{9w}{25}$.

(ii) By considering the equilibrium of the cube and resolving the forces on the cube, show that the reaction between the cube and the horizontal surface is $W+\frac{27w}{125}$;

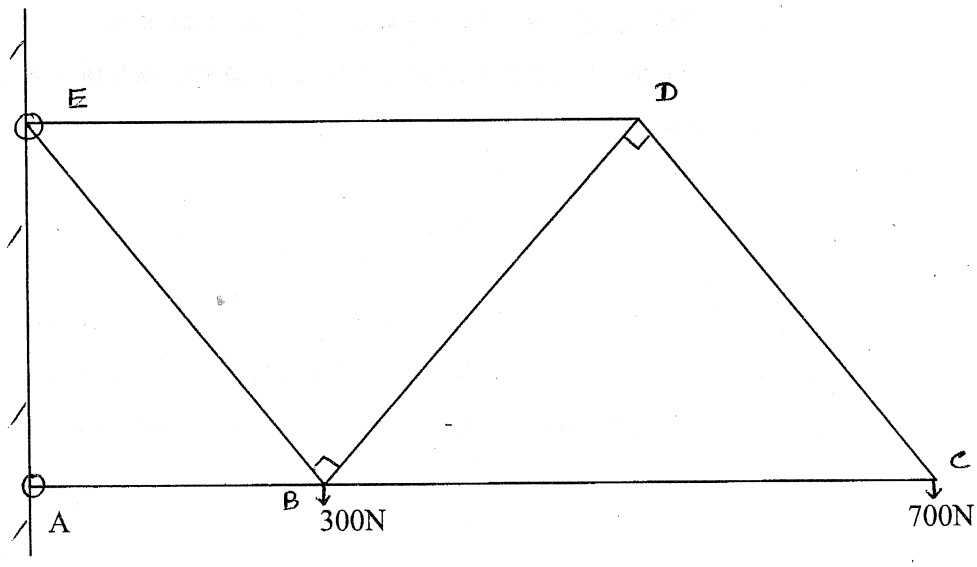
(iii) Find the tension in the string AM.

(iv) Show that $18w \leq 125W$.



07. A uniform sphere of radius a and weight $\frac{w}{\sqrt{3}}$ rest on a rough horizontal table. A uniform rod AB of weight 2W and length 2a is freely hinged at A to a fixed point on table and leans against the sphere so that the centre of the sphere and the rod lie in a vertical plane. The rod makes an angle of 60^0 with horizontal. Show that the frictional force between the rod and the sphere is $\frac{w}{3}$. The coefficient of friction at each point of contact is μ. What is the smallest value of μ which makes equilibrium possible.

08.



The diagram shows a frame work ABCDE, consisting of 6 light rods which are smoothly jointed. The frame work is smoothly hinged to a vertical wall at A and E and the triangles ABE, BDE and BCD are all right angled and isosceles. When loads 300N and 700N are hung from B and C respectively, the framework is in equilibrium in a vertical plane.

(i) Explain why the reaction on the hinge at A is horizontal, and find its magnitude.

(ii) Find the magnitude and the direction of the reaction on the hinge at B.

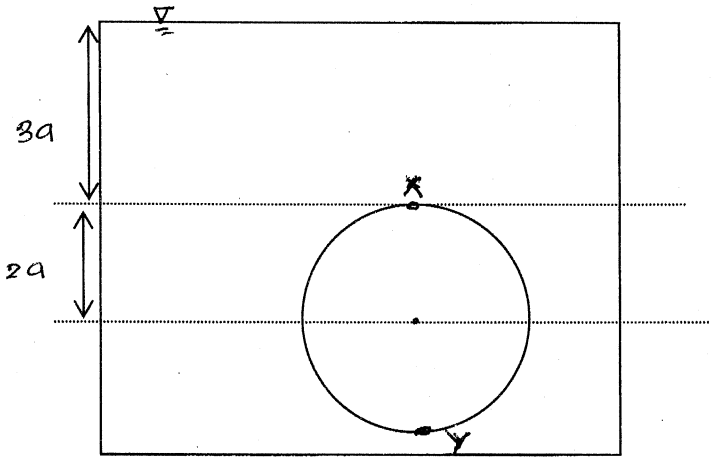
(iii) By drawing a force diagram, find the magnitude of the force acting on each member of the framework and state which rods are in compression.

SECTION – C

09. (a) An alloy of two metals whose relative densities (specific gravity) are 10.50 and 6.50. The alloy weights 2.43 kg in air and 2.13 kg in water. Find the weight of each metal and the ratio of their volumes.

(b) 5l of a liquid of a specific gravity 1.3 is mixed with 7*l* of a liquid of a specific gravity 0.78. If the bulk of the liquid shrinks by 1% on mixing, find the specific gravity (Relative density) of the mixture.
(The density of water = 1000 kgm^{3+})

10. A light flap is used to close a circular hole of radius 2a, in the side of a tank. It is kept in place by bolts at the highest (x) and the lowest (y) points of the hole. The tank is filled with water of density ρ to a height 3a above the highest point (x) of the hole. Find the thrust on the circular flap. Show that the forces on the lower and upper bolts x and y are 11 $\pi a^3 \rho g$, 9 $\pi a^3 \rho g$ respectively. (Assume that the centre of pressure of a vertical circular lamina of radius 'a' immersed with its centre at a depth h (from the water surface) acts at a distance $\frac{a^2}{4h}$ below from its centre.)



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