THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – FOUNDATION (LEVEL 01)

FINAL EXAMINATION - 2007

MPZ 1330– PURE MATHEMATICS II

DURATION – THREE (03) HOURS

Index No.....

DATE : 24th April 2008 **TIME: 9.30 a.m. – 12.30 p.m.**

ANSWER (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION.
YOU CAN'T USE MOBILE PHONES AS A CALCULATOR.
YOU CAN USE NON PROGRAMMABLE CALCULATOR.

SECTION - A

01. (a) Verify the identity $(x^2+4y^2)^2-64x^2y^2 \equiv x^4-56x^2y^2+4y^4$

Hence find the factors of $x^4-56x^2y^2+4y^4$

Given that,

$x^4-56x^2y^2+4y^4=-1679$ and

$x^2+4y^2+8xy = 73$

By using the above factors find the value of x^2+4y^2-8xy

Hence find the values of x^2+4y^2 and xy

Expand and find the values of $(x+2y)^2$ and $(x-2y)^2$ using the above results.

Hence find the values of x and y
{Where x > 0, y > 0 & x > y}

(b) Solve for x; $\sqrt{5-2x}+\sqrt{15+3x}=\sqrt{26-5x}$

(c) Solve the following system of equations.

$2x+3y+4z = 28$

$5y-x+3z = 19$

$2y+x-z = 3$

02. (a) Given that $\rho^x = y \Leftrightarrow \log_p y = x$ By using the above definition, show that

$\log_x y . \log_y x = 1$

Hence show that $\log_8 2008 = \dfrac{1}{\log_{2008} 8}$

Hence find the value of $\dfrac{1}{\log_8 2008} + \dfrac{1}{\log_{251} 2008}$

(b) $4.4^{2x} - 65.4^x + 16 = 0$ By substituting $4^x = t$

Show that $4t^2 - 65t + 16 = 0$

Hence show that $x = -1$ or 2

(c) Simplify (without using calculator or tables) ,

$$\log \frac{133}{65} + 2\log \frac{13}{7} - \log \frac{143}{90} + \log \frac{77}{171}$$

03. (a) Prove that the roots of equation $x^2 - 2ax + a^2 - b^2 - c^2 = 0$ are real; [where a,b,c are real]

(b) For what values of λ will the equation $x^2 - 2(1+3\lambda)x + 7(3+2\lambda) = 0$ has equal roots.

(c) If α and β are the roots of $3x^2 + 4x - 5 = 0$ find the value of

(*I*) $(3\alpha + 4)^{-2} + (3\beta + 4)^{-2}$

(*II*) $(3\alpha + 4)^3 + (3\beta + 4)^3$

SECTION - B

04. (a) Write down the expressions for Sin (A +B), Cos (A +B)

(i) Given that $Sin45^o = Cos45^o = \dfrac{1}{\sqrt{2}}, Cos30^o = Sin60^o = \dfrac{\sqrt{3}}{2}$

$Sin30^o = Cos60^o = 1/2$

Hence prove that $Sin75^o = \dfrac{\sqrt{2} + \sqrt{6}}{4}$.

(ii) Given that θ and ϕ are acute angle such that $Sin\theta = \frac{24}{25}, Cos\phi = \frac{12}{13}$; find, (without using tables or calculator) the exact values of Sin(θ+φ) and Cos(θ+φ) Deduce that $\frac{\pi}{2} < \theta + \phi < \pi$.

(b) Two circles with radii 10cm and 12cm respectively have their centres C_1 and C_2 14 cm apart. By considering the given figure. Prove that

(i) $Sin\theta = 5/6\, Sin\phi$

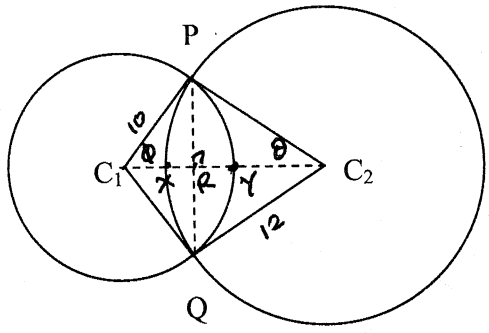
(ii) $PR = 10\, Sin\phi$, $RC_2 = (14 - 10\, Cos\phi)$

(iii) $Cos\phi = 0.5429$ and $\phi = 0.997\, rad$

(iv) $Sin\theta = 0.6999$ and $\theta = 0.775\, rad$

Find (v) The area of the triangle PC_2Q

(vi) The area of the triangle PC_1Q
(vii) The area of the sector $PXQC_2$
(viii) The area of the sector $PYQC_2$
(ix) The area common to both circles



05. (a) Prove the following identities

(i) $\frac{\cot\theta}{1+\cos ec\,\theta} - \frac{\cot\theta}{1-\cos ec\,\theta} = 2\sec\theta$

(ii) $(1+\tan\theta)^2 + (1-\tan\theta)^2 = 2\sec^2\theta$

(iii) $\dfrac{\cos\theta+\cot\theta}{\tan\theta+\sec\theta} \equiv \cos\theta\cot\theta$

(iv) $(1+\sec\theta+\tan\theta)(1+\cos ec\theta+\cot\theta) \equiv 2(1+\tan\theta+\cot\theta+\sec\theta+\cos ec\theta)$

(v) $\dfrac{\cot\alpha+\tan\beta}{\cot\beta+\tan\alpha}=\cot\alpha\tan\beta$

(b) If K = Tan 25°

Express $\dfrac{Tan205^o - Tan115^o}{Tan245^o + Tan335^o}$ in terms of K.

06. (a) Sketch the graph of $y = \cos x$ where $0 \le x \le 2\pi$ Hence sketch the graph $y = 3\cos 2x\ ; -\pi \le x \le \pi$. Draw the graph of $y = 3Sin\left(\dfrac{\pi}{2}+2x\right)+1; -\pi \le x \le \pi$

Mark the maximum and minimum points of the graph and give the coordinates of the maximum and minimum points.

(b) A line DE paralled to the base BC of a triangle ABC cuts AB,AC in D and E respectively. The circle which passes through D, and thouches AC at E, meets AB at F. Prove that F,E,C,B lie on a circle.

SECTION - C

07. (a) Evaluate the following limites

(i) $x \xrightarrow{lt} 0 \dfrac{(1+x)^{1/3}-1}{(1+x)^{1/2}-1}$ (ii) $x \xrightarrow{lt} \dfrac{\pi}{2} \dfrac{Tanx - Sinx}{x(1-\cos x)}$

(b) Find the differential coefficient (derivatives) of the following with respect to x.

(i) $y = \cos^{-1}\left(\dfrac{4+5\cos x}{5+4\cos x}\right)$ (ii) $y = \sqrt{\dfrac{1-\sin x}{1+\sin x}}$

(c) If $y = a e^{mx} + b e^{-mx}$, prove that

$$\frac{d^2y}{dx^2} - m^2 y = 0 ;$$

Hence deduce that $\frac{dy_1}{dx}$ and $\frac{d^2y_1}{dx^2}$ where $y_1 = -a e^{-mx} + be^{mx}$

08. Given that $y = f(x) = x^3 - 6x^2 + 9x + 15$ find the values of x for which $\frac{dy}{dx} = 0$. Hence determine the nature of these stationary values of y. Sketch the curve $y = f(x) = x^3 - 6x^2 + 9x + 15$ given that $f(-0.95) \cong 0$ Hence, deduce the graphs of the curves

(a) $y = g(x) = -x^3 + 6x^2 - 9x - 15$

(b) $y = h(x) = \dfrac{1}{x^3 - 6x^2 + 9x + 15}$

(c) $y = k(x) = (x-1)^3 - 6(x-1)^2 + 9(x-1) + 1$

09. (a) Integrate the following with respect to x

(i) $\int \cos 8x \sin 6x \, dx$ (ii) $\int \cos^3 x \sin^5 x \, dx$

(iii) $\int \dfrac{dx}{(x+4)^2 + 3^2}$ (iv) $\int \dfrac{dx}{\sqrt{5^2 - (x-1)^2}}$

(b) Evaluate the following integrals.

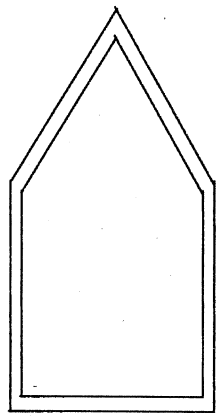
(i) $\int_0^{\pi/2} \dfrac{\cos x}{1 + \sin^2 x}$ (ii) $\int_0^3 \sqrt{9 - x^2}\, dx$

(iii) $\int_0^{\pi/3} \left(\dfrac{-\sin x + 7\cos x - 5}{4\cos x + 3\sin x - 5} \right) dx$

(c) Show that $\int_0^{\infty} \dfrac{dx}{(x^2 + a^2)(x^2 + b^2)} = \dfrac{\pi}{2ab(a+b)}$

10. (a) Draw a rough sketch and find the area of region bounded by two curves $y^2 = 8x$ and $x^2 = 8y$

(b) The section of a corner window is a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16m find the width of the window in order that maximum light may be admitted.



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