THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
DIPLOMA IN TECHNOLOGY – LEVEL 4
FINAL EXAMINATION – 2008

MEX4272 – VIBRATION AND FAULT DIAGNOSIS

DATE : 12 th MARCH 2009
TIME : 0930 HRS. – 1230 HRS.
DURATION : THREE (03) HOURS

ANSWER ANY FIVE (05) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 01

Fig. Q1 shows an inverted pendulum consisting of a spherical ball of radius R and mass m_2 and a uniform rod of mass m_1 and length a. Pendulum is pivoted at O. Two springs, each of stiffness k, are attached to the ball as shown in the figure. Determine the equation of motion and derive an expression for the natural frequency of vibrations of the system. Take the moment of inertia of a sphere about its centre as $\frac{2}{5}mr^2$, where m is the mass and r is the radius of the sphere.

If $m_1 = m$, $m_2 = \frac{m}{5}$ and $R = \frac{a}{4}$, show that the natural frequency ω_n is approximately given by

$$\omega_n = 0.438\sqrt{25\frac{k}{m} - 6\frac{g}{a}}.$$

What is the condition for stability of the system?

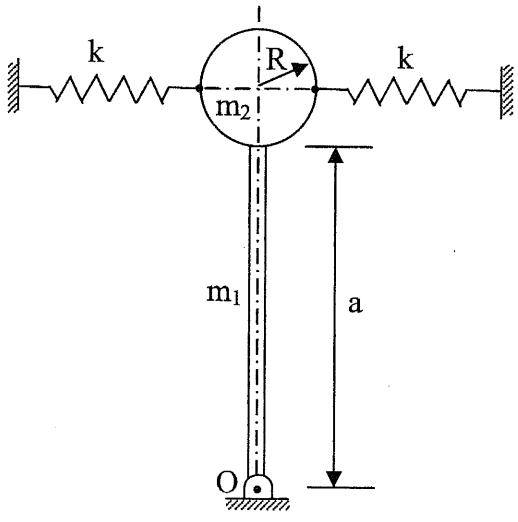


Fig. Q1

QUESTION 02

Fig. Q2 shows a spring-mass-damper system where the mass m is supported on a spring of stiffness k and a damper of damping constant b. Determine the equation of motion and hence derive expressions for the free response of the system (motion of the mass denoted by x) for the two cases, (i) damping factor $\zeta < 1$ (ii) damping factor $\zeta > 1$, and explain the nature of the responses. Please note that $\zeta = \dfrac{b}{2m\omega_n}$ and $\omega_n = \sqrt{\dfrac{k}{m}}$

If m = 16 kg, k = 1600 N/m, determine the free response for the cases:

(i) b = 256 Ns/m and

(ii) b = 480 Ns/m

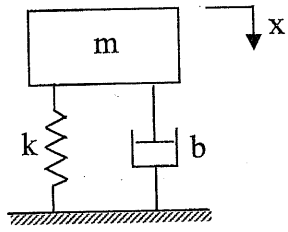


Fig. Q2

QUESTION 03

Two rods of negligible mass each carrying a single concentrated mass m are supported on two springs, each having a stiffness k, as shown in Fig. Q3. The system is free to oscillate in the vertical plane and the rods are horizontal in the equilibrium position. Determine the equations of motion and the frequency equation for small angular oscillations of the rods.

Also determine the natural frequencies of angular oscillations in terms of k and m.

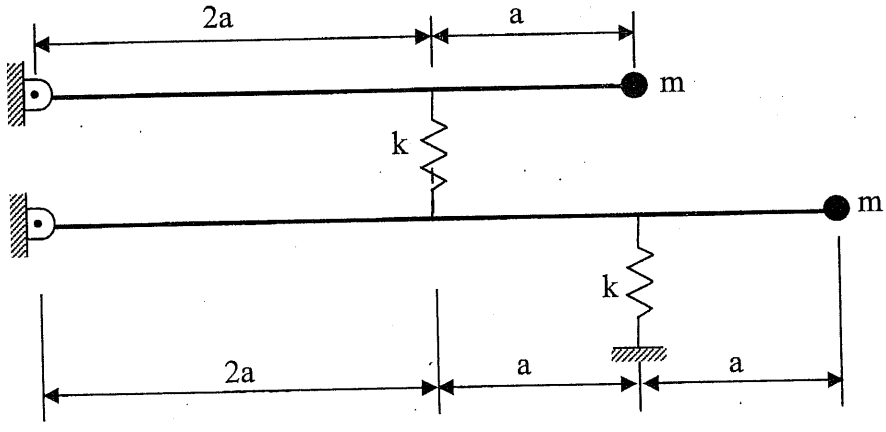


Fig. Q3

QUESTION 04

Fig. Q4 shows a spring-mass system in which m_1 and m_2 are the masses and k_1 & k_2 are the stiffnesses of the springs. Determine the equations of motion using Newton's laws of motion. Verify your result by the Lagrange method.

If $m_1 = m$, $m_2 = 4m$, $k_1 = k$ and $k_2 = 2k$, show that the frequency equation can be expressed as $\omega^4 - 1.75\left(\frac{k}{m}\right) + 0.5\left(\frac{k}{m}\right)^2 = 0$, where ω is the natural frequency of vibration.

Also determine the natural frequencies in terms of k and m.

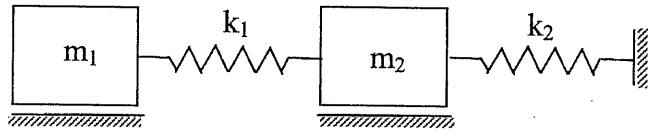


Fig. Q4

QUESTION 05

Fig. Q5(a) shows a flexible rotor system consisting of two rotors each of moment of inertia I and two shafts each of torsional stiffness K. Determine the natural frequencies of free torsional oscillations of the system.

Fig. Q5(b) shows a one degree of freedom system consisting of a rotor of moment of inertia I_R and a shaft of torsional stiffness $2K$. If this torsional system has the same frequency as the lower value of the frequency of the system shown in Fig. Q5(a), determine the moment of inertia of the rotor I_R in terms of I.

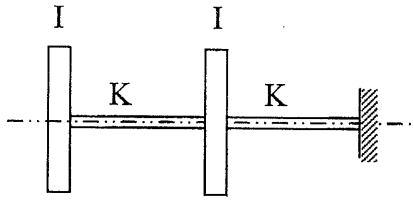


Fig. Q5(a)

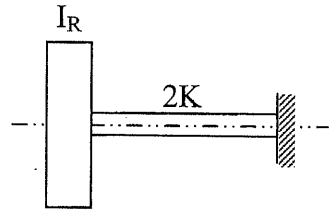


Fig. Q5(b)

QUESTION 06

The unit step response of a control system is given by $y(t) = 2 - e^{-t}(2 \cos t + \sin t)$. Determine the following.

(i) Transfer function of the system

(ii) Input-output differential equation of the system.

(iii) Damped natural frequency, undamped natural frequency and the damping ratio.

(iv) Response of the system to a unit impulse input.

(v) Steady state response for a unit step input?

You may assume that all initial conditions are zero.

QUESTION 07

A dynamic model of a fluid coupling system is shown in Fig. Q7. The fluid coupler is represented by a rotary viscous damper with damping constant b. It is connected to a rotary load of moment of inertia I restrained by a torsional spring of stiffness K as shown. Obtain the frequency transfer function of the system relating the restraining torque T of the spring (output) to the angular displacement excitation $\theta(t)$ (input) that is applied to the far end of the fluid coupler. If $\theta(t) = \theta_o \sin \omega t$, what is the magnitude of T at steady state.

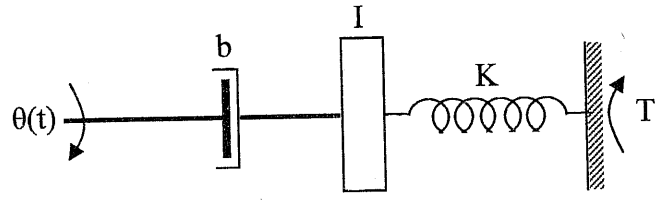


Fig. Q7

QUESTION 08

(i) Briefly explain the procedure adopted in carrying out vibration monitoring of equipment.

(ii) Briefly explain the relative advantages and disadvantages of the three basic maintenance strategies used in maintaining machinery and equipment.

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-.....-f^{n-1}(0)$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

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